# THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

## (U.G.—CCSS)

### Core Course—Mathematics

### MM 3B 03—CALCULUS

Time : Three Hours             Maximum : 30 Weightage

I. Answer *all* questions :—

1. Let $F(t) = 2(t - 1) + 3$. Evaluate F at the input value $x + 2$.

2. If $f(x) = \sqrt{x}$ and $g(x) = x + 1$ find $(f \circ g) x$.

3. Find the domain and range of $f(x) = 1 + x^2$.

4. Evaluate $\displaystyle\lim_{x \to -2} \frac{2x - 4}{x^3 + 2x^2}$.

5. At what points are the function $y = \dfrac{1}{x - 2} - 3x$ is continuous.

6. State Rolle's theorem.

7. What are the critical points of $f$ given

$$f'(x) = (x - 1)(x + 2)(x - 3).$$

8. Evaluate $\displaystyle\lim_{x \to \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

9. Find $dy$ if $y = \dfrac{2x}{1 + x^2}$.

10. Find the intervals in which the function $f$ is increasing. Given $f'(x) = x(x - 1)$.

11 The length of the longest sub interval of a partition is called its ————.

12 Evaluate $\int_0^{\pi/3} 2 \sec^2 x \, dx$.

(12 × ¼ = 3 weigh~~

II. Answer all *nine* questions.

13 Find the volume of the solid generated by revolving the region bounded by the lines ~
$x = 2$ and the curve $y = x^3$.

14 Find $\dfrac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$.

15 Find the average value of $f(x) = -3x^2 - 1$ on $[0, 1]$.

16 Evaluate $\sum_{k=1}^{6} \left(3 - k^2\right)$.

17 Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 0$.

18 Find the absolute maximum and minimum values of $f(x) = -x - 4, -4 \le x \le 1$.

19 Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through th~
$(0, 2)$.

20 Find the work done by a force of $F(x) = \dfrac{1}{x^2}$ N along the $x$-axis from $x = 1$ m. to $x = 1$~

21 Evaluate $\int_0^{\pi/4} \tan x \sec^2 x \, dx$.

(9 × 1 = 9 wei~

**III.** **Answer any *five* questions :—**

22 Find the lateral surface area of the cone generated by revolving the line segment $y = \dfrac{x}{2}$, $0 \le x \le 4$, about the y-axis.

23 Find the length of the curve $y = \tan x$, $\dfrac{-\pi}{3} \le x \le 0$.

24 Find the asymptotes of the curve $y = \dfrac{x+3}{x+2}$.

25 Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

26 Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

27 Find the intervals on which the function $g(t) = -t^2 - 3t + 3$ is increasing and decreasing.

28 About how accurately should we measure the radius $r$ of a sphere to calculate the surface area $S = 4\pi r^2$ within 1 % of its true value.

(5 × 2 = 10 weightage)

**IV.** **Answer any *two* questions :—**

29 Find the length of the curve $y = \dfrac{4\sqrt{2}}{3} x^{3/2} - 1$, $0 \le x \le 1$.

30 Show that the centre of mass of a straight, thin strip or rod of constant density has halfway between its two ends.

31 State and prove the fundamental theorem of calculus.

(2 × 4 = 8 weightage)