16U213 (Pages: 2) Name.....
Reg. No.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY-2017
(Regular/Supplementary/Improvement)
(CUCBCSS – UG)
CC15UMAT2C02 – MATHEMATICS
(Complementary Course:Mathematics)
(2015 Admission Onwards)

Time: Three Hours Maximum: 80 Marks

PART A

Answer **All** Questions. Each question carries 1 mark

1. Write $\sinh^{-1} x$ as a logarithmic function.
2. The derivative of sech $2x$ with respect to x is .....
3. Investigate the convergence of $\int_1^{\infty} \frac{dx}{x^2}$
4. Find the nth term of the sequence $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots\ldots\ldots$
5. Find $\lim_{n\to\infty} \left(1+\frac{3}{n}\right)^n$.
6. State whether the series $\sum_{n=1}^{\infty} \frac{n+1}{n}$ converge or diverge.
7. Find $\frac{\partial f}{\partial x}$ at $(4,-5)$ if $f(x,y) = x^2 + 3xy + y$.
8. Define a power series about $x = a$.
9. Find the nth partial sum of $\sum_{n=1}^{\infty}(-1)^n$.
10. Find the Cartesian coordinates of the cylindrical coordinates $(0,1,0)$.
11. Find the polar equation of the circle $x^2 + (y-3)^2 = 9$.
12. What is the domain of $(x,y) = \sqrt{y - x^2}$?

$(12 \times 1 = 12$ *Marks*$)$

PART B

Answer any **NINE** Questions. Each question carries 2 marks

13. Prove that $\cosh^2 x - \sinh^2 x = 1$ using the definition of $\sinh x$ and $\cosh x$.
14. Find the volume of the solid generated by revolving the region bounded by the line $x + 2y = 2$ and the lines $y = 0, x = 0$ about the x-axis.
15. Examine the convergence of the improper integral $\int_{-\infty}^{0} \cosh x \, dx$.
16. Find the perimeter of the cardioid $r = a(1 - \cos\theta)$.
17. Evaluate $\int \frac{\sinh x}{\cosh^4 x} dx$.
18. Show that the sequence $0, 2, 0, 2, \ldots\ldots\ldots 0, 2, \ldots\ldots$ does not converge to zero.
19. Evaluate $\lim_{n\to\infty} \frac{\sin^2 n}{2^n}$
20. Find the sum of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3}{2^n}$
21. Prove that the following alternating series is convergent.
$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots\ldots\ldots + \frac{(-1)^{n+1}}{n} + \ldots\ldots \infty$$
22. Find the domain and range of the function $f(x,y) = \sin^{-1}(y - x)$.

23. Find the linearization of $f(x, y) = x^2 + y^2 + 1$ at the point $(1,1)$.
24. Evaluate $\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x}$

$(9 \times 2 = 18$ *Marks*$)$

PART C

Answer any **SIX** Questions. Each question carries 5 marks

25. Show that $\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$ for all real $x \geq 1$.
26. Find the area of the surface generated by revolving about the x-axis, the portion in the first and second quadrants of the circle $x^2 + y^2 = a^2$.
27. Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
28. Verify $w_{xy} = w_{yx}$, if $w = e^x + x \ln y + y \ln x$.
29. Show that the function $f(x, y) = \frac{x}{\sqrt{x^2+y^2}}$ has no limit as $(x, y) \to (0,0)$.
30. Find the Maclaurin's series for $f(x) = \ln(1 + x)$. Also find the Taylor polynomials of orders 0,1,2 and 3 generated by f at zero.
31. Evaluate $\int_0^1 \frac{2dx}{\sqrt{3+4x^2}}$
32. Compare $\int_1^{\infty} \frac{dx}{x^2}$ and $\int_1^{\infty} \frac{dx}{1+x^2}$ with limit comparison test.
33. Graph the lemniscate $r^2 = 4 \sin 2\theta$.

$(6 \times 5 = 30$ *Marks*$)$

PART D

Answer any **TWO** Questions. Each question carries 10 marks

34. Find
    a) The area of the surface of the solid formed by the revolution of the cardioid $r = a(1 + \cos\theta)$ about the initial line.
    b) The spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.
35. Using chain rule express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ in terms of r and θ if $w = \tan^{-1}\left(\frac{y}{x}\right), x = r\cos\theta$, $y = r\sin\theta$. Also evaluate $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ at $\left(1, \frac{\pi}{6}\right)$.
36. Show that the following series converges to $\tan^{-1} x$ for all $-1 < x < 1$

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots\ldots\ldots$$

$(2 \times 10 = 20$ *Marks*$)$

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