16U228 (Pages: 2) Name..................

Reg. No.................

SECOND SEMESTER B. Sc DEGREE EXAMINATION, MAY-2017

(Regular/Supplementary/Improvement)

(CUCBCSS – UG)

CC15U ST2C 02-PROBABILITY DISTRIBUTIONS

(Complementary Course: Statistics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 80 Marks

Section A

*(One word questions. Answer **all** questions. Each question carries 1 mark)*

Fill up the blanks:

1. If $\mu_{11} = -6,\ \mu_{02} = 4\ and\ \mu_{20} = 9$ are the bivariate central moments of two random variables X and Y, then the correlation between X and Y is $r_{xy} =$ ....
2. If the moment generating function of a random variable is $(\frac{1}{3} + \frac{2}{3}e^t)^{10}$, then its variance is ....
3. Mean of the uniform distribution defined over the interval (0, 1) is ....
4. The p.d.f. of standard Cauchy distribution is ....
5. The coefficient of variation of Poisson distribution with mean 4 is ....

Write true or false

6. Characteristic function exists for all distributions.
7. The sum of two independent Binomial variates is also a Binomial variate. The result holds for the difference also.
8. Gamma distribution tends to the Normal distribution for large value of parameter λ.
9. The measure of skewness, $\beta_1 = 0$ for a Normal distribution.
10. Linear combination of independent Normal variates is also a Normal variate.

($10 \times 1 = 10$ marks)

Section B

*(One sentence questions. Answer **all** questions. Each question carries 2 marks)*

11. Show that $Var\ (aX) = a^2 Var(\ X)$, where a is a constant.
12. Define m.g.f. of a random variable.
13. Define conditional variance.
14. Define uniform distribution of the discrete type.
15. What is the third central moment of a Poisson distribution with parameter λ.
16. Define Pareto distribution.
17. Define convergence in probability.

(7×2=14 marks)

Section C

(*Paragraph questions. Answer any **three** questions. Each question carries **4** marks*)

18. If a random variable X has the p.d.f. $f(x) = \frac{1}{2} e^{-|x|}$, $-\infty < x < \infty$, find its m.g.f.
19. The joint p.d.f. of a pair (X, Y) of two random variables is given by
$f(x, y) = \frac{x+y}{21}$, x = 1, 2, 3 and y = 1, 2. Find the conditional distribution of X given Y = 2.
20. Determine the binomial distribution for which mean is 4 and variance 3 and find its mode.
21. Point out any two specific situations where Poisson distribution can be applied. Show that the mean and variance of a Poisson random variable are equal.
22. If E(X) = 3, $E(X^2) = 13$, use Chebyschev's inequality to find a lower bound for P (-2 < X< 8). **(3 x 4 = 12marks)**

Section D

(*Short Essay questions. Answer **any four** questions. Each question carries **6** marks*)

23. A random variable X has the probability mass function $p(x) = \frac{1}{(2^x)}$, x = 1, 2, 3, ...
Find its mean and variance.
24. Two random variables X and Y have the joint p.d.f.
$$f(x,y) = \begin{cases} k(4-x-y); 0 \leq x \leq 2; 0 \leq y \leq 2 \\ 0, \ otherwise \end{cases}$$
Find (i) the constant k, (ii) the marginal density functions of X and Y.
25. Establish the lack of memory property of exponential distribution.
26. State the chief characteristics of the normal distribution.
27. State and prove the Bernoulli's law of large numbers.
28. State the Lindberg- Levy form of central limit theorem stating the assumptions clearly.

(4x6=24 marks)

Section E

(*Essay questions. Answer any **two** questions. Each question carries **10** marks*)

29. Define expected value of a random variable and prove that for any two random variables X and Y for which expectations exist (i) E(X + Y) = E(X) + E(Y); and E(XY) = E(X) E(Y), if X and Y are independent.
30. X and Y have a bivariate distribution given by
$$f(x,y) = \begin{cases} \frac{2x+y}{30} & ,where \ (x,y) = (1,1),(1,2),(1,3),(2,1),(2,2),(2,3) and \\ 0 \ elsewhere. \end{cases}$$
Find the conditional mean and variance of X given Y = 1.
31. For a binomial distribution, with usual notation , show that
$\mu_{r+1} = pq[\frac{d\mu_r}{dp} + nr\mu_{r-1}]$
32. Let $\{X_n\}$ be a sequence of mutually independent and identically distributed random variables with mean μ and finite variance σ^2. If $S_n = X_1 + X_2 + ... + X_n$, prove that the law of large numbers does not hold for the sequence $\{S_n\}$.

(2×10 = 20 marks)

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