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Name..........................................

Reg. No...................................

# FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(U.G.–CCSS)

Core Course—Mathematics

## MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours

Maximum : 30 Weightage

## Section A

*Answer* **all** *questions.*
*Each question carries ¼ weightage.*

1. What is the order of $(y''')^2 + (y'')^7 + y = \sin t$ ?

2. Give the general form of a separable equation.

3. Test for exactness : $(2x + 4y)\,dx + (2x - 2y)\,dy = 0$.

4. Solve $y'' - y = 0$.

5. State Abel's theorem.

6. Find the Wronskian of $y_1 = e^{2t}$, $y_2 = e^{-3t}$.

7. What is $L\{e^{-at}\}$ ?

8. State true *or* false : The Laplace transform is a linear operator.

9. Find $L\{\sin 3t\}$.

10. What is the fundamental period of $\sin 7\,t$ ?

11. What is the heat conduction equation ?

12. Is the function $f(x) = x|\,x\,|$ even, odd or neither ?

(12 × ¼ = 3 weightage)

**Turn over**

## Section B

*Answer all questions.*
*Each question carries 1 weightage.*

13. Verify that $y = 3t + t^2$ is a solution of $ty^1 - y = t^2$.

14. State the existence and uniqueness theorem for first order initial value problems.

15. Verify whether 'y' is an integrating factor of $ydx + 2xdy = 0$.

16. Solve $2y'' - 5y' + 3y = 2e^{4t}$.

17. Solve $y'' + a^2 y = 0$.

18. Find $L\{\sinh 7t\}$.

19. Show that convolution is commutative.

20. Show that the sum of two even functions is even.

21. Graph the square wave function.

$(9 \times 1 = 9 \text{ weig}$

## Section C

*Answer any five questions.*
*Each question carries 2 weightage.*

22. Find an integrating factor and solve :

$$\left(2x^2 + y\right) dx + \left(x^2 y - x\right) dy = 0.$$

23. Solve the initial value problem :

$$(y+2) dx + y(x+4) dy = 0 ; y(-3) = -1.$$

24. Solve the initial value problem :

$$y'' - 2y' + y = te^{t+4}, y(0) = 1, y'(0) = 1.$$

25. Solve $y'' + y = \sin t \sin 2t$.

26. Find $L^{-1}\{2/(s^2 + 3s - 4)\}$.

27. Find $L\{t^2 e^{at}\}$.

28. Find the Fourier sine series of $f(x) = \begin{cases} x, 0 \le x < 1, & f \text{ is of} \\ 1, 1 \le x < 2, & \text{period 4} \end{cases}$.

$(5 \times 2 = 10 \text{ wei}$

## Section D

*Answer any* **two** *questions.*
*Each question carries 4 weightage.*

29. Solve by the method of variation of parameters :

$$y'' + y = \tan t, \, 0 < t < \frac{\pi}{2}.$$

30. (i) Using convolution, find $L^{-1}\left\{\dfrac{1}{s^2(s+2)}\right\}$.

    (ii) Using Laplace transforms, solve    $y'' - y' - 6y = 0$

$$y(0) = 1, y'(0) = -1.$$

31. Find the Fourier series expansion of :

$$f(x) = \begin{cases} -x, -2 \le x < 0, \\ x, 0 \le x < 2 \end{cases} f(x+4) = f(x)$$

    Deduce that $\dfrac{\pi^2}{8} = 1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots$

<div align="right">(2 × 4 = 8 weightage)</div>