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Reg. No..

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2014

(U.G.–CCSS)

Core Course—Mathematics

MM 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 30 Weightage

I. Answer *all* questions :

1 Plane through $P_0(x_0, y_0, z_0)$ and normal to $\vec{m} = Ai + Bj + Ck$ is ——————.

2 Find the parametric equation for the line through the points $P(-3,2,-3)$ and $Q(1,-1,4)$.

3 Vector equation for the line through $P_0(x_0, y_0, z_0)$ and parallel to $\vec{V}$ is $\overline{P_0P} =$ ——————.

4 A vector function $\vec{r}(t)$ is continuous at a point $t = t_0$ in its domain if $\lim_{t \to t_0} \vec{r}(t) =$ ——————.

5 Domain of the function $w = \sin(xy)$ is the entire plane. Then range = ——————.

6 $\lim_{(x,y) \to (0,0)} \dfrac{3x^2 - y^2 + 5}{x^2 + y^2 + 2} =$ ——————.

7 Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ if $f(x,y) = (x^2 - 1)(y + 2)$.

8 Find the gradient of $g(x,y) = y - x^2$ at (–1, 0).

9 The curl of a vector field $\vec{F} = Mi + Nj$ at the point (*x*, *y*) is ——————.

10 Curvature of a straight line is ——————.

11 Define Saddle point.

12 Examine whether $F = yi + (x + z)j - yk$ conservative.

(12 × ¼ = 3 weightage)

Turn over

II. Answer all *nine* questions :

13 Find the angle between the planes :

$3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

14 Find the spherical co-ordinate equation for the sphere :

$x^2 + y^2 + (z-1)^2 = 1$.

15 Show that $\bar{u}(t) = (\sin t)i + (\cos t)j + \sqrt{3}\,k$ is orthogonal to its derivative.

16 Find the equation for the plane through P_0 (0, 2,–1) and normal to $\bar{n} = 3i - 2j - k$.

17 Find the acceleration of a moving particle at $t = 1$ whose position vecto

$\bar{r}(t) = (t+1)i + (t^2 - 1)j$.

18 Find the parametric equation for the line that is tangent to the curve :

$\bar{r}(t) = (a\sin t)i + (a\cos t)j + bt\text{K}$ at $t_0 = 2\pi$.

19 If $t_0 = 0$ find the arc length parameter along the helix $\bar{r}(t) = (\cos t)i + (\sin t)j + tk$.

20 Write the range of the function $f(x,y) = xy$.

21 State Stoke's theorem.

(9 × 1 = 9 weight

III. Answer any *five* questions :

22 Find T and N for the plane curve :

$\bar{r}(t) = (2t+3)i + (5-t^2)j$.

23 Find the point where the line $x = 1 + 2t, y = 1 + 5t, z = 3t$ intersects the plane $x + y +$

24 Find the distance from the point S (1, 1, 5) to the line L : $x = 1 + t, y = 3 - t, z = 2t$.

25 Find the curvature for the space curve $\bar{r}(t) = (e^t \cos t)i + (e^t \sin t)j + 2k$.

26 Calculate the outward flux of the field $F(x, y) = xi + y^2 j$ across the square bounded b

lines $x = \pm 1, y = \pm 1$.

27 Evaluate $\int_C (xy + y + z)\,dz$ along the curve $\vec{r}(t) = 2ti + tj + (2 - 2t)k, 0 \le t \le 1$.

28 Find the area enclosed by the lemiscate $r^2 = 4\cos 2\theta$.

(5 × 2 = 10 weightage)

V. Answer any *two* questions :

29 Find the plane determined by the intersecting lines :

$L_1 : x = -1 + t,\ y = 2 + t, z = 1 - t, -\infty < t < \infty$

$L_2 : x - 1 - 4s, y = 1 + 2s, z = 2 - 2s, -\infty < s < \infty$

30 Find an upper bound for the magnitude of the error E in the approximation :

$f(x,y,z) \approx L(x,y,z)$ over the rectangle R. Given $f(x,y,z) = xz - 3yz + 2$ at P_0 (1, 1, 2).

$R : |x - 1| \le 0.01, |y - 1| \le 0.01, |z - 2| \le 0.02$.

31 Show that $F = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$ is conservative and find a potential function for it.

(2 × 4 = 8 weightage)