Spectral Domain Modeling of the Effect of Film Purity for Superconducting Slotline

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***Abstract*—A computation technique is presented to analyse the electromagnetic response of High Temperature Superconducting (HTS) film purity using a slotline. The complex conductivity of HTS films for three different material purity is modeled using Coffey-Clem (CC) unified theory. By using the surface impedance calculated by CC model, the dyadic Green's functions in Spectral Domain Method (SDM) are formulated for the slotline. The Galerkin's procedure is employed for the computation of the propagation parameters when HTS slotlines of different purities are guiding a fixed frequency microwave signal. Changes in the propagation parameters are analyzed for the variation of the film thickness at different working temperatures. The paper presents a modeling method for studying the microwave response of the HTS film purity.**

***Index Terms*—Electromagnetic propagation, Green's function methods, Transmission lines, High Temperature Superconductors.**

I. INTRODUCTION

The High Temperature Superconducting (HTS) films of different purity are manufactured using various techniques and are reported in the literature [1], [2]. Material purity of the HTS film used as the thin strip on inhomogeneous transmission lines significantly changes the complex conductivity and the propagation parameters [3]. Based on the experimental data regarding purity, YBCO thin films can be broadly classified into three types [3]. The high purity films, the medium purity films, and the low purity films are differentiated based on the phenomenological description of complex penetration depth. In this paper we present a modeling methodology to study the relationship between HTS film purity with electromagnetic wave propagation using a slotline for a wide range of temperature and strip thickness.

The modeling of HTS transmission lines without considering the vortex effects when no dc magnetic field is applied may appear straight forward [3]. But HTS films like YBCO are extreme type II materials and they will be in mixed state even at very low magnetic field values [4]. The microwave signal itself may make them in vortex state. At liquid nitrogen temperatures, the HTS films will have significant vortex activities like flux creep and flux flow [5]. For the accurate computation of conductivity of the HTS films and the propagation parameters of the slotline we need to employ a theory which takes into account the fields due to microwave signal, applied dc field and vortex generated fields [5]–[7]. Accordingly, the modeling of inhomogeneous HTS transmission lines are quite challenging. Again, the inhomogeneous structure of striplines make their field derivations and formulation of dyadic Green's functions mathematically complicated [8]–[13]. The HTS contribution of the slotline have to be properly incorporated into the dyadic Green's functions for making the HTS film purity analysis accurate [14].

The HTS microstrip line, resonator, slotline and Coplanar waveguide have been modeled and their propagation parameters have been analyzed in our earlier works [9], [10], [12]–[15]. In this work we study the impact of HTS film purity on the microwave signal propagation by incorporating flux dynamics which include creep effect, flux flow phenomenon and vortex pinning and present a computation technique to specifically analyse this film purity effect on the wave propagation. We take a slotline with HTS material of varying film purity on a substrate like sapphire and place it in an applied dc magnetic field. For the sake of simplicity, we neglect the anisotropic natures of the permittivity of the substrate and the conductivity of the HTS material.

II. MODELING TECHNIQUE

In Spectral Domain Method (SDM), by matching the electromagnetic boundary conditions of the different interfaces of the slotline, we obtain the fourier transformed coupled equation as $[\tilde{\mathbf{Y}}][\tilde{\mathbf{E}}] = [\tilde{\mathbf{J}}]$ where $\tilde{Y}$'s are the admittance elements, $\tilde{E}$'s are the unknown electric fields and $\tilde{J}$'s are the strip currents. The detailed discussion of the method is available elsewhere [8]. To incorporate the HTS contribution, we expand the unknown electric field as $\tilde{E} = \tilde{E}^e + Z_s\tilde{J}$ where $\tilde{E}^e$ is the Electric field distribution at the interface excluding the region of the strip, Z_s is the surface impedance due to HTS material and $\tilde{J}$ represent the strip current. Accordingly, our algebraic equation will be modified as $[\tilde{\mathbf{Y}'}][\tilde{\mathbf{E}}] = [\tilde{\mathbf{J}}]$ where $\tilde{Y}'$'s are the modified admittance Green's functions [10]. The propagation parameters are computed using Galerkin procedure.

The Coffey-Clem (CC) model gives a self-consistently determined penetration depth $\tilde{\lambda}(\omega, B, T)$ in terms of field dependent penetration depth λ, normal fluid skin depth δ_{nf} and the complex effective skin depth $\tilde{\delta}_{\text{vc}}$ [5] as

$$\tilde{\lambda}(\omega, B, T) = \left(\frac{\lambda^2 - (\mathrm{i}/2)\tilde{\delta}_{\text{vc}}^2}{1 + 2\mathrm{i}\lambda^2\delta_{\text{nf}}^{-2}}\right)^{1/2} \tag{1}$$

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The response of the superfluid is given by $\lambda(B,T)$ and is defined as $\lambda(B,T) = \lambda(0,T)/[1 - B/B_{c2}(T)]^{1/2}$, where $\lambda(0,T) = \lambda_0/[1-(T/T_c)^{\gamma}]^{1/2}$. The temperature-dependent upper critical field is defined as $B_{c2}(T) = B_{c2}(0)[1 - (T/T_c)^2][1 + (T/T_c)^2]^{-1}$. The length associated with the vortex response is given as $\tilde{\delta}^2_{\mathrm{vc}}(\omega,B,T) = 2\tilde{\rho}_{\nu}/\mu_0\omega$ where the effective resistivity $\tilde{\rho}_{\nu}(\omega,B,T) = B\phi_0\tilde{\mu_{\nu}}(\omega,B,T)$ and $\tilde{\mu_{\nu}}(\omega,B,T)$ is the complex dynamic vortex mobility. Since the problem of vortex dynamics is similar to that of the response of an ionic conductor in a potential well, CC model uses a dynamic mobility which is based on the Schneider's continued-fraction expansion function which is truncated to give the dc response of the vortex mobility [5]. In doing so we can exactly reproduce the Ambegaokar and Halperin result [5]. This mobility term is given as

$$\tilde{\mu_{\nu}}(\omega,B,T) = \frac{1}{\eta}\left(1 + \left(\frac{\mathrm{i}\omega\eta}{\xi\kappa_{\rho}} + \frac{1}{I_0^2(\nu)-1}\right)^{-1}\right)^{-1} \qquad (2)$$

Here η is the viscous drag coefficient defined by $\eta = B\phi_0/\rho_{\mathrm{f}}$ and where $\rho_{\mathrm{f}} = \rho_{\mathrm{n}}B/B_{\mathrm{c2}}(T)$ is the BS (Bardeen-Stephen) flux flow resistivity. The force constant of the pinning potential well appearing in the Eq. (2) is given by $\kappa_{\mathrm{p}} = \kappa_{\mathrm{p0}}[1-(T/T_{c2})^2]^2$ where T_{c2} is the temperature at which $B = B_{c2}(T)$. We take $\xi = I_1(\nu)/I_0(\nu)$, where I_0 and I_1 are the modified Bessel functions of the first kind of order zero and one respectively, and the argument is defined by $\nu = U_0(B,T)/2k_{\mathrm{B}}T$. The temperature and field dependent barrier height of the periodic potential is given by $U_0(B,T) = U[1-(T/T_{c2})^{3/2}]B^{-1}$ [9], [10]. The third length in the Eq. (1) is associated with the normal-fluid and is given as $\delta^2_{\mathrm{nf}}(\omega,B,T) = 2\rho_{\mathrm{nf}}/\mu_0\omega$ where the normal fluid resistivity, ρ_{nf}, is expressed as $\rho_{\mathrm{nf}} = \rho_{\mathrm{n}}/f(T,B)$ where $f(T,B) = 1 - [1-(T/T_c)^{\gamma}][1 - B/B_{c2}(T)]$.

For thin strip cases [11], the complex surface impedance of the superconductor Z_{s} is given as $Z_s = h/\tilde{\sigma}$, where h is the superconducting strip thickness and $\tilde{\sigma}$ is complex conductivity. The rf complex resistivity appearing in the expression $E = \tilde{\rho}J$, is related to $\tilde{\lambda}$ via $\tilde{\rho} = \mathrm{i}\mu_0\omega\tilde{\lambda}^2$ with $\tilde{\sigma} = 1/\tilde{\rho}$ [5].

III. RESULTS AND DISCUSSION

Computation is performed by taking the characteristics values of YBCO though our transmission line model is independent of any particular sample. Here we take T_c = 92 K, $\rho_n(T) = 1.1 \times 10^{-8}T + 2 \times 10^{-6}\Omega$ m, U = 0.15 eV , κ_{p0} = 2.1×10^4 N/m and B_{c2} = 112 T [5], [6]. The experimental data for high purity film is taken as λ_0 = 144.7 nm, $\gamma = 2$ [1] whereas for medium purity film the corresponding values are 219 nm and 1.68 [2]. For low purity film, we take the values given as in Vendik *et. al.* as 250 nm and 1.5 (Table II of [3]).

The structural parameters of the slotline are taken as following. The thickness of the superconducting strip h is varied from 0.2 μm to 0.3 μm. Sapphire, with permittivity value 9.8, is taken as substrate with thickness d as 2.0 mm.

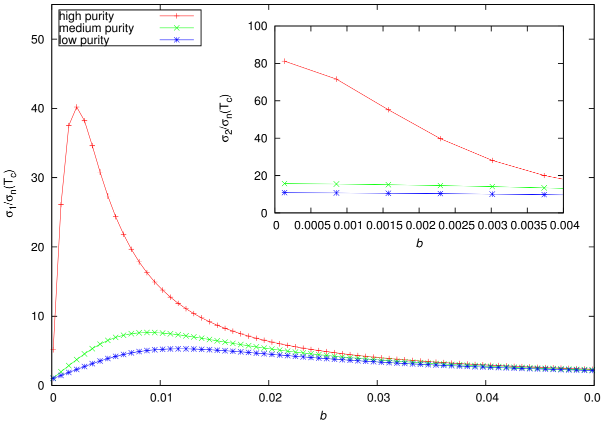

Fig. 1. The variation of the normalized real part of the complex conductivity $(\sigma_1/\sigma_n(T_c))$ for three quality films with respect to the reduced field b at reduced temperatures t= 0.9 for the frequency 33 GHz. The inset gives the normalized imaginary part of complex conductivity $(\sigma_2/\sigma_n(T_c))$

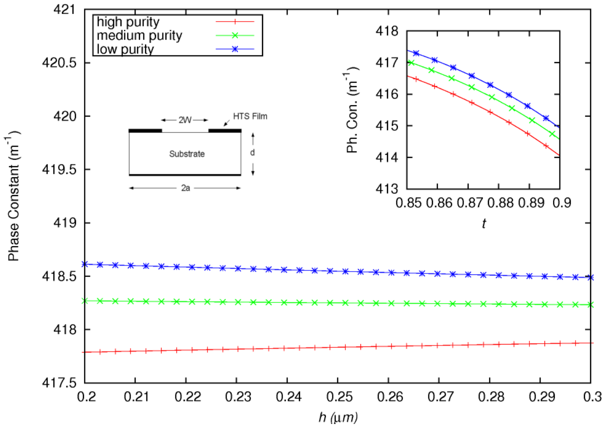

Fig. 2. The variation of the phase constant with the superconducting strip thickness h for three quality films. The right inset shows the variation of phase constant with reduced temperature t. The left inset shows the layout of slotline with superconducting strip.

The width of the slot W for the slotline is 1.0 mm and that of the substrate a is 6.0 mm. The inset of the Fig. 2 gives the layout of the HTS slotline. For applying the strip line equation of Pond [11] we take $h < \lambda(B,T)$. The two other constraints imposed by the CC model are $\lambda(B,T) \gg a_0$ and $h \gg a_0$ [5], where a_0 is the intervortex spacing, and the above constraints are satisfied in all our simulations.

In Fig. 1 we plot $\tilde{\sigma}$ with respect to the reduced field $b = B/B_{c2}(0)$ at a reduced temperatures $t = T/T_c$ = 0.9 for the frequency 33 GHz. For high purity film, the normalized real part of complex conductivity $(\sigma_1/\sigma_n(T_c))$ shows a peaking phenomena which is not observable with medium or low purity HTS film. This peaking of σ_1 may be used as a criterion for high purity HTS film. The normalized imaginary part of conductivity $(\sigma_2/\sigma_n(T_c))$ for high purity film is manifold greater than other films at low magnetic field. As b increases, all the values converge together as expected due to the increase of vortices.

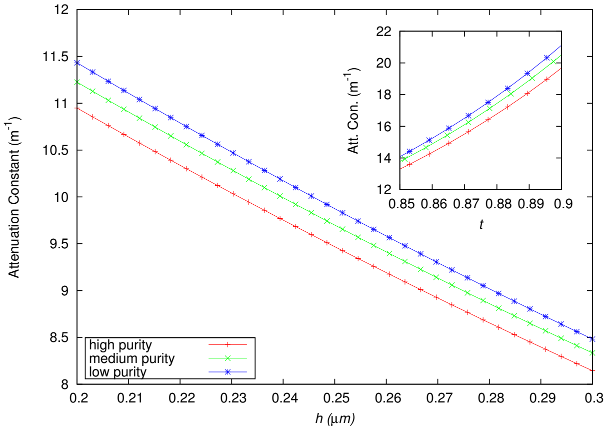

Fig. 3. The variation of the attenuation constant with h for three quality films. The inset shows the variation of attenuation constant with t.

In Fig. 2 and Fig. 3 we plot the variation of the phase constant and attenuation constant of the three types of films by varying the strip thickness h at the reduced temperature $t = 0.85$ and dc field value 2.8 T. The signal frequency is taken at 10 GHz. In the inset, their corresponding changes with the temperature are plotted. Signal dissipation is lower for the high quality film as is evident from the lower value of attenuation constant. As the temperature increases, the greater flux creep and flux flow movements will again enhance the attenuation (inset of Fig. 3). Phase constant is relatively constant with respect to h and t for all the three films.

IV. CONCLUSION

A modeling technique is presented to study the electromagnetic response of the purity of HTS films using a slotline. The real and the imaginary parts of the complex conductivity of high purity HTS film shows differentiating behaviour from other samples. The variation in the propagation parameters due to film purity is also studied.

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