D 90908 (Pages : 3) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(U.G.—CCSS)

Core Course—Mathematics

MM 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*

1. Is $\frac{dy}{dx} + x^2 y = \frac{1}{y}$ linear ?
2. Find an integrating factor of $xdy - ydx = 0$?
3. Does $\frac{dy}{dt} = e^t$ basic a solution passing through $(0, 1)$?
4. Find the solution of $y'' - 4y' + 4y = 0$.
5. Find the differential equation whose solution is $y = c_1 e^{kx} + c_2 e^{-kx}$.
6. Are x and x^2 linearly independent.
7. What is the Laplace Transform of sinh at ?
8. State the shifting properly of Laplace Transforms.
9. Define step function.
10. Give the wave equation.
11. Is the function $f(x) = x^2 \cos nx$ even.
12. If $f(x)$ is an odd function, the coefficient of cosines in the Fourier series expansion of $f(x)$ is ————.

(12 × ¼ = 3 weightage)

Part B

Answer **all** *questions.*

13. Solve $x\left(1+y^2\right)dx + y\left(1+x^2\right)dy = 0$.
14. Show that $\mu(x) = x$ is an integrating factor of $\left(x^2 - 2x + 2y^2\right)dx + 2xy\,dy = 0$.

Turn over

15. State the existence and uniqueness theorem for solution of a first order differential equation

16. $y'''' + 8y'' + 16y = 0$ – Solve this.

17. Solve $y'' - y' - 6y = 20e^{-2x}$.

18. Find $L\left\{e^{-7t}\cos 3t\right\}$.

19. Find $L^{-1}\left\{\frac{1}{(s+9)^3}\right\}$.

20. Find a_o for the periodic function (of period 2π) : $f(x)\begin{cases}-k, & -\pi < x < 0 \\ k & 0 < x < \pi.\end{cases}$

21. Define the convolution integral and show that it is commutative.

(9 × 1 = 9 weig

Part C

Answer any **five** *questions.*

22. Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$.

23. (i) Define linear and non-linear first order differential equations with examples.

(ii) When is a differential equation said to be exact ? Derive a necessary and sufficient co for $Mdx + Ndy = 0$ to be exact.

24. Show that $y = c_1 x + c_2 x^2$ is the general solution of $x^2y'' - 2xy' + 2y = 0$ on any inter containing zero and find the particular solution for which $y(1) = 3$ and $y'(1) = 5$.

25. Find the general solution by the method of variation of parameters : $y'' - 2y' + y = 2x$.

26. Using the method of Laplace Transforms, solve $y'' - 3y' + 2y = 4e^{2t}$, $y(0) = -3$, $y'(0) = 5$.

27. Using convolution properly, show that $L^{-1}\left\{\frac{1}{(s-2)(s-3)}\right\} = e^{3t} - e^{2t}$.

28. Show that $u = e^{nx+iny}$ and $u = e^{nx-iny}$ are both solutions of $u_{xx} + u_{yy} = 0$.

(5 × 2 = 10 we

Part D

*Answer any **two** questions.*

29. Solve $\left(D^2 + 2D + 5\right)y = x + \sin 2x$.

30. Find the Fourier series of $f(x) = |x|$ in $[-\pi, \pi]$ and deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots = \frac{\pi^2}{8}$.

31. Solve by the method of separation of variables : $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.

$(2 \times 4 = 8$ weightage$)$