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Reg. No..................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(U.G.—CCSS)

Core Course—Mathematics

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 30 Weightage

I. Objective type questions : Answer all *twelve* questions :

1 Let $f(x) = \frac{3x}{x+1}$ for $x \in A = \{x \in R : x \neq -1\}$. Then range of f is ———.

2 Using algebraic properties of R, prove $a \cdot b = 6 \Rightarrow a = 1$.

3 State completeness property of R.

4 Write the supremum of $S = \left\{\frac{1}{n} ; n \in N\right\}$.

5 Give an example of a convergent sequence (x_n) of positive numbers with $\lim \frac{x_{n+1}}{x_n} = 1$.

6 Give an example of a Cauchy sequence.

7 State True *or* False. "Every bounded sequence is convergent".

8 If (x_n) and (y_n) are two sequences, such that $x_n < y_n$ and $\lim x_n = x$; $\lim y_n = y$. What is the relation between x and y ?

9 Give an example of a monotonic sequence.

10 State True *or* False. "Every open interval in an open set".

11 Prove that $z\bar{z} = |z|^2$.

12 Write the multiplicative inverse of the non-zero complex number $z = x + iy$.

(12 × ¼ = 3 weightage)

Turn over

II. Very short answer questions. Answer all *nine* questions :

13 Let $f : A \to B ; g : B \to C$ be functions. Show that if gof is injective then f is injective.

14 Use mathematical induction to prove that $n^3 + 5n$ is divisible by 6.

15 Define $\in$-neighbourhood of $a \in R$.

16 Find the supremum and infimum of the set $S = \left\{1 - \frac{(-1)^n}{n} ; n \in N\right\}$.

17 If $S \subseteq T \subseteq R$, where $S \neq \phi$, then show that if T is bounded above then $\text{Sup } S \leq \text{Sup } T$.

18 "A sequence in R can have atmost one limit"—Prove.

19 Using definition of limit, prove that $\lim \left(\frac{1}{n}\right) = 0$.

20 Prove that a Cauchy sequence in bounded.

21 Find arg of z where $z = \frac{i}{-2-2i}$.

(9 × 1 = 9 weigh

III. Short answer questions. Answer any *five* questions :

22 State and prove Bernoulli's inequality.

23 If $a, b \in R$, prove that $|a+b| \leq |a| + |b|$.

24 Let A and B be bounded non-empty subsets of R and $A + B = (a + b ; a \in A, b \in B)$. Prove Sup (A + B) = Sup A + Sup B.

25 State and prove Squeeze theorem.

26 If a sequence $X = (x_n)$ of real numbers converges to a real number x, then prove that subsequence $X' = (x_{nk})$ also converges to x.

27 Show that z is either real or purely imaginary iff $(\bar{z})^2 = z^2$.

28 Locate the points in the camplex plane for which $|z-1| = |z+i|$.

(5 × 2 = 10 weightage)

V. Essay questions. Answer any *two* questions :

29 (a) Prove that the set Q of all rational numbers is denumerable.

(b) Suppose S and T are sets such that $T \subseteq S$. Prove that z + T is infinite, then S is infinite.

30 (a) Prove that the union of arbitrary collection of open subsets is R is open.

(b) Give an example to show that the arbitrary intersection of open set is not open.

31 (a) If z_1 and z_2 are two non-zero complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then prove that $z_1 = -z_2$.

(b) Evaluate $\sqrt{1-\sqrt{3}\,i}$.

(2 × 4 = 8 weightage)