

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

(CUCBCSS—UG)

Complementary Course

MAT 3C 03—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type Questions)*Answer all twelve questions.*

1. Write the general form of Bernoulli's differential equation.
2. Find the solution of the differential equation $y' = \frac{-y}{x}$.
3. What is the order of the differential equation $y'' - (y')^3 + 4 = 0$?
4. State Cayley Hamilton theorem.
5. What is the rank of a $(n \times n)$ non-singular matrix ?
6. Write the normal form of the matrix : $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
7. Write the parametric equation of the curve $\frac{x^2}{4} + \frac{y^2}{3} = 1$.
8. Define Irrotational vector.
9. Find curl v , where $v = [2y, 5x, 0]$.
10. Find the tangent to the curve $r(t) = ti + t^3 j$ at $(1, 1, 0)$.
11. Define scalar potential of a vector.
12. State Gauss's divergence theorem.

(12 × 1 = 12 marks)

Part B (Short Answer Type Questions)*Answer any nine questions.*

13. Find the orthogonal trajectories of the family of curves $y = ce^{-x}$.
14. Write the condition for the differential equation $Mdx + Ndy = 0$ become exact. What is the form of its solution ?

Turn over

15. Find the integrating factor of the linear differential equation $y' - y = e^{2x}$.
16. Find characteristic roots of the matrix :
$$\begin{bmatrix} -1 & 2 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$
.
17. Write the elementary transformations in a matrix.
18. Find the component of vector $a = [4, 2, 0]$ in the direction of $b = [1, -1, 2]$.
19. Find the directional derivative of $f = xyz$ at the point $P(-1, 1, 3)$ in the direction of $i - 2j + 2k$.
20. Find the unit normal to the level surface $z^2 = 4(x^2 + y^2)$ at the point $P(1, 0, 2)$.
21. Find $\text{div } v$, where $v = xyzi + 3zxi + zk$.
22. Define Jacobian.
23. Find value of λ if $a = [4, 2, \lambda]$ and $b = [2, -3, 1]$ are orthogonal.
24. Write the formula for finding the area of a plane region as a line integral over the boundary.

(9 × 2 = 18 marks)

Part C (Short Essay Type Questions)*Answer any six questions.*

25. Solve the initial value problem $y' + y \tan x = \sin 2x$, $y(0) = 1$.
26. Solve $xy' = y + 3x^4 \cos^2\left(\frac{y}{x}\right)$.
27. Find the eigenvalues and eigenvector corresponding to any one eigenvalue of the matrix : $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{bmatrix}$.
28. Use Cayley Hamilton theorem to find A^{-1} and A^4 , where $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$.
29. Find the tangential and normal components of acceleration of an object moving along the curve $r(t) = e^t i + e^{-t} j$.

30. Find tangent to the ellipse $\frac{1}{4}(x^2 + y^2)$ at the point $P\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.
31. Find the area of the cardioid $r = a(1 - \cos \theta)$, $0 \leq \theta \leq 2\pi$.
32. Evaluate the double integral $\iint_R y^2 dx dy$ where R is the region bounded by the unit circle in the first quadrant.
33. Verify Green's theorem in the plane for the vector $F = (y^2 - 7y)i + (2xy + 2x)j$ and the region bounded by the $x^2 + y^2 = 1$.

(6 × 5 = 30 marks)

Part D (Essay Type Questions)*Answer any two questions.*

34. Test for consistency and solve the following system of equation.

$$\begin{array}{ll} x + y + z + 3 = 0 & 3x + 26y + 2z = 9 \\ \text{(a) } 3x + y - 2z + 2 = 0 & \text{(b) } 5x + 3y + 7z = 4 \\ 2x + 4y + 7z - 13 = 0. & 7x + 2y + 10z = 5. \end{array}$$

35. (a) Solve the differential equation :

$$2 \sin(y^2) dx + xy \cos(y^2) dy = 0, \quad y(2) = \sqrt{\frac{\pi}{2}}.$$

- (b) Prove that $\text{Curl}(\text{grad}f) = 0$.

36. Verify Stokes's theorem for $F = [y, z, x]$ over the surface of the paraboloid $z = 1 - (x^2 + y^2)$, $z \geq 0$.

(2 × 10 = 20 marks)