D 11164 (Pages : 4) Name..................................

Reg. No..................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

Fill in the blanks :

1. The brachistochrone problem was first solved by ———.
2. Write the heat equation for a rod of finite length completely as a boundary value problem.
3. Find the general solution of $y'' - y = 0$.
4. Find the Laplace transform of $\cosh(2\,at)$.
5. Write the formulas for computing the Fourier coefficients in the Fourier series expansion of a periodic function $f(x)$ of period $2L$.
6. Define an exact differential equation. Is $(x+y)dy - (x-y)dx = 0$ exact ? Why ?
7. Solve the system : $\frac{dx}{dt} = y,\ \frac{dy}{dt} = x$.
8. Define unit step function and write its Laplace transform.
9. Give an example of a non-linear differential equation in the dependent variable y and the independent variable x of second order.
10. Show that $u(x, y) = f(x - ay) + g(x + ay)$ is a solution of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{dy^2}$.

Turn over

11. Show that sum and product of two even functions are even functions.

12. Compute the Wronskian of the functions e^t and e^{-t}.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*

Each question carries 4 marks..

13. $\left(x + e^{-y/2}\right)\dfrac{dy}{dx} = 2,\ y(0) = 0.$

14. Use Laplace transform to find the solution of $\dfrac{dy}{dt} = t,\ y(0) = 1.$

15. Using convolution find the inverse Laplace transform of $\dfrac{1}{(s-2)(s-1)}$.

16. Show that any separable equation $M(x) + N(y)y' = 0$ is also exact.

17. Solve : $t^2y'' + ty' + y = 0.$

18. Use method of variation of parameters to solve : $y'' + 4y = 3\operatorname{cosec} t$.

19. Given that $y_1(t) = t^{-1}$ is a solution of $2t^2y'' = 3ty - y = 0, t > 0$. Find a fundamental set of solutions.

20. If $f(x) = x, -\pi \le x \le \pi$ is a 2π-periodic function, find a_n, the coefficient of $\cos(nx)$ in its Fourier series expansion.

21. Find the values of a and b such that the equation $(ax + by)\dfrac{dy}{dx} = bx + ay$ is exact and hence solve it.

22. Find the Laplace transform of the function :

$$f(t) = \begin{cases} 2, & \text{if } 0 < x < \pi \\ 0, & \text{if } \pi < x < 2\pi \\ \sin t, & \text{if } x > 2\pi \end{cases}$$

23. State the conditions for the convergence of a Fourier series of a 2π periodic function.

24. Transform the equation $u'' + 0.5\,u' + u = 0$ into a system of first order differential equations.

25. Show that Wronskian of the fundamental solutions of $y'' + y = 0$ is actually non-zero.

26. Write the conditions for the existence of the Laplace transform of a function.

(10 × 4 = 40 marks)

Section C

Answer any* six *out of nine questions.

Each question carries 7 marks.

27. Solve:

(a) $(3x+4y)\dfrac{dy}{dx} = 2x + y,\ y(0) = 0.$

(b) $y - y' = 2xy, y(0) = 1.$

28. Find an integrating factor for the equation $(3xy + y^2) + (x^2 + xy)y' = 0$ and solve it.

29. Find the general solution of $y'' - 2y' + y = 2\cos(2t) - t^2$.

30. Find the Fourier cosine series expansion of $f(x) = \sin\left(\pi x/_{L}\right)$ when $0 < x < L$.

31. Find :

(a) $\mathcal{L}(\cosh(at)\cos(at))$.

(b) $\mathcal{L}^{-1}\left(\dfrac{1}{(s^2+\omega^2)^2}\right)$.

32. Solve the boundary value problem using Laplace transform : $y'' - y = 1$, where $y(0) = 0, y\left(\pi/_{2}\right) = 1.$

33. State and prove Abel's theorem.

34. Prove the convolution theorem for Laplace transform.

35. (a) Solve using the method of separation of variables : $\frac{\partial u}{\partial x} = a^2 \frac{\partial u}{\partial y}$, $u(x, 0) = 1$, $u(0, y) = -1$

(b) Solve : $y'' + y' + y = 2t$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 marks.

36. Find the Fourier series of :

$$f(x) = \begin{cases} k, & \text{if } -\pi/2 < x < \pi/2 \\ 0, & \text{if } \pi/2 < x < 3\pi/2 \end{cases},$$

assuming it is period 2π and deduce that $\pi/4 = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$

37. Find the solution of the initial value problem $y'' - 2y - 1 = 0$, $y(0) = 0$, $y'(0) = 1$ in two ways ; one of them must be using Laplace transforms.

38. Derive the wave equation by stating the assumptions involved and find its D'Alembert's solution.

(2 × 13 = 26 marks)