D 11163 (Pages : 3) Name..............................

Reg. No..............................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS—UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *the* **twelve** *questions.*
Each question carries 1 *mark.*

1. Define a countable set.
2. What do you mean by trichotomy law of real numbers ?
3. State Bernoulli's inequality.
4. Find all x satisfying $|x-1|<|x|$.
5. State the completeness property of the set of real numbers.
6. What are the conditions for a subset of real numbers to be an interval ?
7. If $a>0$ find $\lim\left(\frac{1}{1+na}\right)$.
8. State Squeeze theorem for limit of sequences.
9. Give the divergence criteria for a sequence of real numbers.
10. Find $Arg(z)$ if $z=-1-i$.
11. Define contractive sequence.
12. Find the exponential form of $\left(\sqrt{3}-i\right)^6$.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of* **fourteen** *questions.*
Each question carries 4 *marks.*

13. Verify that the set of all integers $\mathbb{Z}$ is denumerable.
14. If $a\geq 0$ and $b\geq 0$, prove that $a<6$ if and only if $a^2<b^2$.
15. State and prove arithmetic-geometric mean inequality.

Turn over

16. Define infimum of a set. If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, prove that inf (S) = 0.

17. If $t > 0$ prove that there is an n_t in $\mathbb{N}$ such that $0 < \frac{1}{n_t} < t$.

18. State and prove the betweenness property of irrational numbers.

19. Determine the set A of all x satisfying $|2x + 3| < 7$.

20. Test the convergence of the sequence (x_n) if $x_n = \frac{\sin n}{n}$.

21. Define Cauchy sequence. Find a sequence (x_n) which is not Cauchy such that $\lim |x_n - x_{n+1}| = 0$.

22. Prove that every convergent sequence of real numbers is a Cauchy sequence.

23. Show that subsequence of a converging real sequence always converge to the same limit.

24. State and prove Bolzano-Weierstrass theorem.

25. Find all values of $(-27\,i)^{\frac{1}{3}}$.

26. Prove that $|z_1 - z_2| \geq \| z_1 | - | z_2 \|$ for all $z_1, z_2 \in \mathbb{C}$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of* **nine** *questions.*
Each question carries 7 marks.

27. Show that the unit interval $[0, 1]$ is uncountable.

28. Prove that there is a real x whose square is 2.

29. If A is any set, prove that there is no surjection of A on to the set $\mathcal{P}(A)$ of all subsets of A. Deduce that power set of natural numbers is uncountable.

30. If $I_n = [a_n, b_n], n \in \mathbb{N}$ is a nested sequence of closed and bounded intervals, prove that there is a real number which lies in I_n for all n.

31. Sate and prove monotone convergence theorem for a sequence.

32. Show that every contractive sequence is convergent.

33. Discuss the convergence of the following (x_n) where (i) $x_n = \left(1 + \frac{1}{2n}\right)^n$; (ii) $x_n = \sum_{m=1}^{n} \frac{1}{m!}$.

34. State Cauchy's convergence criterion. Use it to test the convergence of $x_n = \sum_{m=1}^{n} \frac{1}{m}$.

35. Find the square roots of $\sqrt{3} + i$ and express them in rectangular form.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of* **three** *questions.*
Each question carries 13 marks.

36. (a) State and prove the characterization theorem for intervals.

(b) Show that between any two real numbers there is a rational number.

37. (a) State and prove the ratio test for the convergence of real sequences.

(b) If a > 0 construct a sequence of real numbers which will converge to the square root of a.

38. (a) Let $X = (x_n)$ and $Y = (y_n)$ be real sequences that converge to x and y respectively. Prove the following :

(i) $\lim(x_n + y_n) = x + y$.

(ii) $\lim(x_n - y_n) = x - y$.

(iii) $\lim(x_n y_n) = xy$.

(iv) $\lim(cx_n) = cx, c \in \mathbb{R}$.

(b) Discuss the convergence of $\frac{n!}{n^n}$.

(2 × 13 = 26 marks)