

**SECOND SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2015**

(CUCBCSS—UG)

Core Course—Mathematics

MAT 2B 02—CALCULUS

Three Hours

Maximum : 80 Marks

**Part A**

Answer all the **twelve** questions.  
Each question carries 1 mark.

1. Evaluate  $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$ .
2. Find the intervals in which the function  $f$  is increasing given  $f^1(x) = x^{-1/3}(x+3)$ .
3. State the Mean Value Theorem.
4. What are the critical points of  $f$  given  $f'(x) = (x-1)(x+2)(x-3)$ .
5. Find  $dy$  if  $y = \sin 3x$ .
6. Evaluate  $\int_0^4 \left( 3x - \frac{x^3}{4} \right) dx$ .
7. The length of the longest subinterval of a partition is called its \_\_\_\_\_.
8. Write the sums without sigma notation and then evaluate the sum  $\sum_{k=1}^2 \frac{6k}{k+1}$ .
9. If  $\int_0^3 f(x) dx = 5$  find  $\int_0^3 \sqrt{2} f(x) dx$ .
10. A function with a continuous first derivative is said to be \_\_\_\_\_.
11. The radius  $r$  of a circle increases from  $r_0 = 10$  m to 10.1 m. Estimate the increase in the circle's area  $A$  by calculating  $dA$ .
12. If  $f$  is smooth in  $[a, b]$  then the length of the curve  $y = f(x)$  from  $a$  to  $b$  is  $L =$  \_\_\_\_\_.

(12 × 1 = 12 marks)

Turn over

## Part B

Answer any **nine** questions.  
Each question carries 2 marks.

13. Find the work done by a force of  $F(x) = \frac{1}{x^2}$  N along the  $x$ -axis from  $x = 1$  m to  $x = 10$  m.

14. Find the absolute maximum and minimum values of  $f(x) = 4 - x^2$ ,  $-3 \leq x \leq 1$ .

15. Evaluate  $\int_0^{2\pi} \frac{\cos z}{\sqrt{4 + 3 \sin z}} dz$ .

16. Find the volume of the solid generated by revolving the region bounded by the lines  $x = 1$  and the curve  $y = x^3$ .

17. Evaluate  $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t dt$ .

18. Show that if  $f$  is continuous on  $[a, b]$ ,  $a \neq b$  and if  $\int_a^b f(x) dx = 0$  then  $f(x) = 0$  at least one point in  $[a, b]$ .

19. Evaluate  $\sum_{k=1}^6 (3 - k^2)$ .

20. Find the linearization of  $f(x) = \sqrt{1+x}$  at  $x = 3$ .

21. Find the average value of  $f(x) = x^2 - 1$  on  $[0, \sqrt{3}]$ .

22. About how accurately should we measure the radius  $r$  of a sphere to calculate the surface area  $S = 4\pi r^2$  within 1% of its true value.

23. Find the length of the curve  $x = \sin y$ ,  $0 \leq y \leq \pi$ .

24. Find the area of the region enclosed by the parabola  $y = 2 - x^2$  and the line  $y = -x$ .

## Part C

30

Answer any **six** questions.  
Each question carries 5 marks.

25. Find the length of the curve  $y = \tan x$ ,  $-\frac{\pi}{3} \leq x \leq 0$ .
26. Find the volume of the solid generated by revolving the region bounded by  $y = \sqrt{x}$  and the lines  $y = 1$ ,  $x = 4$  about the line  $y = 1$ .
27. Find the area of the region enclosed by the curve  $y = 2x - x^2$  and the line  $y = -3$ .
28. Find the lateral surface area of the cone generated by revolving the line segment  $y = \frac{x}{2}$ ,  $0 \leq x \leq 4$  about the  $y$ -axis.
29. Find the asymptotes of the curve  $y = \frac{x^2 - 3}{2x - 4}$ .
30. Express the solution of the following initial value problem as an integral :  
Differential equation :  $\frac{dy}{dx} = \tan x$ .  
Initial condition :  $y(1) = 5$ .
31. Find the intervals on which the function  $g(t) = -t^2 - 3t + 3$  is increasing and decreasing.
32. Find the local maxima and local minima of  $g(x) = -x^3 + 12x + 5$ ,  $-3 \leq x \leq 3$ .
33. Find the area between  $y = \sec^2 x$  and  $y = \sin x$  from 0 to  $\frac{\pi}{4}$ .

(6 × 5 = 30 marks)

Turn over

## Part D

Answer any **two** questions.  
Each question carries 10 marks.

34. Show that the centre of mass of a straight, thin strip or rod of constant density has half between its two ends.
35. A rectangle is to be inscribed in a semi-circle of radius 2. What is the largest area then rectangle can have and what are its dimensions?
36. Find the area of the region between the curve  $y = 4 - x^2$ ,  $0 \leq x \leq 3$  and the  $x$ -axis.

(2 × 10 = 20 marks)