15U407 (Pages:2) Name:.......................

Reg. No.......................

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2017

(CUCBCSS - UG)

Mathematics - Complementary Course

CC15U MAT4 C04 – Mathematics

(2015 Admission)

Time: Three Hours Maximum: 80 Marks

PART A

Answer **All** Questions. Each question carries **1** mark.

1. State the superposition principle of homogeneous ordinary differential equations.
2. Apply $D^2 + 2D$ on $sin\ x$.
3. The general solution of Euler Cauchy equation having auxiliary equation with double root m = 3 is ....
4. Find $L(t^m),\ m > 0$.
5. Define Dirac's Delta function.
6. $L^{-1}\left(\frac{1}{(s-2)^5}\right) =$.......................
7. What is the period of $f(x) = e^x$
8. Is $f(x) = x|x|, \forall x \in \mathbb{R}$, an even or odd function?
9. Solve $u_{xx} = u$.
10. State whether true or false. If $f(x)$ is odd, $|f(x)|$ and $f^2(x)$ are even functions.
11. The upper bound of error estimate for $\int_0^1 x\,dx$ using Trapezoidal rule where $n = 10$ is..............
12. Find y_1 using Picard's iteration method to the IVP $y' = 2, y(0) = 0$.

(12 x 1 = 12 Marks)

PART B

Answer **any nine** Questions. Each question carries 2 marks.

13. Solve the IVP $y'' - k^2y = 0, (k \neq 0), y(0) = 1, y'(0) = 1$.
14. Using Wronskian, verify the linear independence of x^5, x^{-5}.
15. Find a differential equation $y'' + ay' + by = 0$ with basis e^{-x}, e^{-2x}
16. Find $L(2t^3 + cosh4t)$.
17. Find $L^{-1}\left(\frac{e^{-3s}}{s^3}\right)$.
18. Define Convolution of f and g. State Convolution theorem for Laplace Transforms.
19. Prove that the sum of two even functions is even.

20. Represent $f(x) = x^2, 0 \leq x \leq \pi$ by a Fourier sine series.

21. Verify that $u = \frac{y}{x}$ satisfies the Poisson equation.

22. Define extension of functions and find an extension of $f(x) = x^2, 0 \leq x \leq \pi$.

23. Solve using Euler's method $\frac{dy}{dx} = 1 - y, y(0) = 0$ at $x = 0.2$.

24. Compute $\int_0^1 x^2 dx$ by rectangular rule with $h = 0.5$.

(9 x 2 = 18 Marks)

PART C

Answer **any six** Questions. Each question carries five marks.

25. Find the curve $y(x)$ through the origin for which $y" = y'$ and the tangent at the origin is $y = x$.

26. Find the general solution of $y" + y = 3x^2$.

27. Solve $(x^2D^2 - 2xD + 2)y = x^3 cosx$ by the method of variation of parameters.

28. Find $L^{-1}\left(\frac{1}{s(s+2)^3}\right)$.

29. Solve the integral equation $y(t) = 1 + \int_0^t y(\tau)d\tau$.

30. Express $f(x) = (x-1)^2, 0 \leq x \leq 1$ by Fourier cosine series.

31. Setting $u_x = p$, solve $u_{xy} = u_x$.

32. Using improved Euler's method, find $y(0.2)$ of the IVP $y' = x + 2y,\ y(0) = 1$, $h = 0.1$.

33. Estimate $\int_0^1 \frac{\sin x}{x} dx$ using Simpson's 1/3 rule.

(6 x 5 = 30 Marks)

PART D

Answer **any two** Questions. Each question carries 10 marks.

34. (a) Find a fundamental set of solutions of $2t^2y" + 3ty' - y = 0, t > 0$ given that $y_1(t) = t^{-1}$ is a solution.

(b) Solve $y" - 2y' + y = x + e^x$.

35. (a) Solve using Laplace transform, the IVP $y" - 5y' + 6y = 6u(t-1), y(0) = 0$, $y'(0) = 0$.

(b) Solve using Runge - Kutta method $\frac{dy}{dx} = y, y(0) = 1$ at the point $x = 1$.

36. Find the Fourier series expansion of $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

Also deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

(2 x 10 = 20 Marks)

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