

15U406

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Name:.....

Reg. No.....

**FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2017**

(CUCBCSS-UG)

Mathematics-Core Course

**CC15U MAT4 B04 -THEORY OF EQUATIONS, MATRICES AND VECTOR  
CALCULUS**

(2015 Admission)

Time: Three Hours

Maximum: 80 Marks

**Part A**

Answer **all** questions. Each question carries **1** mark

1. State fundamental theorem of algebra
2. Form an equation whose roots are negatives of the roots of the equation  
$$x^4 - 3x^3 + 5x^2 + x - 3 = 0.$$
3. Give an example of a standard reciprocal equation
4. State Descarte's rule of Signs.
5. The rank of a square matrix of order 3 of which every element is three is .....

6. If the rank of  $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$  is two then rank of  $\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$  is .....

7. Say True or False. Zero is a characteristic root of an invertible matrix.

8. Give the eigenvalues of the matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -4 & 0 \\ 3 & 5 & 6 \end{bmatrix}$ .

9. Find the parametric equations for the line through the origin and parallel to the vector  
 $3i - 4k$ .

10. Find the Cartesian equation of the surface whose equation in cylindrical coordinates is  
 $r^2 + 4z^2 = 16$ .

11. A particle moves along the curve  $x = 3t^2, y = t^2 - 2t, z = t^3$ . Find the velocity at  $t = 1$ .

12. Find the unit tangent vector of the helix  $r(t) = \cos t i + \sin t j + t k$ .

(12 x 1= 12 Marks)

**Part B**

Answer any **Nine** questions. Each question carries **2** marks

13. Form the polynomial equation of fourth degree with rational coefficients, one of whose roots is  $\sqrt{2} + \sqrt{-3}$ .

(1)

Turn Over

14. If  $\alpha, \beta, \gamma$  are the roots of  $x^3 + ax^2 + bx + c = 0$ , find the value of  $\sum \left(\frac{1}{\alpha}\right)$ .

15. Find an equation whose roots are three less than the roots of the equation

$$x^4 - 5x^3 + 7x^2 - 4x + 5 = 0.$$

16. Show that the equation  $x^6 + 3x^2 - 5x + 1 = 0$  has at least four imaginary roots.

17. Prove that interchange of a pair of rows of a matrix does not alter its rank.

18. Write all possible normal forms of a matrix having rank 4.

19. Find  $a, b$  if the rank of the following matrix  $A$  is 2

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & a & b \\ 1 & 1 & -2 & a \end{bmatrix}$$

20. Check the consistency of the system  $x + y + z = 4, 2x + 5y - 2z = 3$ .

21. Prove that the eigenvalues of a diagonal matrix are the same as its diagonal elements.

22. Find the point where the line  $x = \frac{8}{3} + 2t, y = -2t, z = 1 + t$  intersects the plane

$$3x + 2y + 6z = 6.$$

23. The velocity of a particle moving in space is  $\frac{dr}{dt} = (\cos t)i - (\sin t)j + k$ . Find the particles position as a function of  $t$  if  $r = 2i + k$  when  $t = 0$ .

24. Find the length of one turn of the helix  $r(t) = (\cos t)i + (\sin t)j + t k$ .

(9 x 2 = 18 Marks)

### Part C

Answer any Six Questions. Each question carries 5 marks

25. Diminish the roots of the equation  $x^4 - 8x^3 - x^2 + 68x + 60 = 0$  by 2 and hence solve it.

26. If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 - x + 1 = 0$ , find the value of  $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ .

27. Solve  $3x^5 - 10x^4 - 3x^3 - 3x^2 - 10x + 3 = 0$ .

28. Find the rank of the following matrix by reducing it to its normal form

$$A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

29. Show that the system of equations

$$x + 2y - 3z - 4w = 6, \quad x + 3y + z - 2w = 4, \quad 2x + 5y - 2z - 5w = 10$$

is consistent and hence solve them.

(2)

30. Find the eigenvalues and the corresponding eigenvectors of the matrix  $\begin{bmatrix} -3 & 0 \\ 5 & -1 \end{bmatrix}$ .
31. Find the plane through  $P_0(2, 1, -1)$  and perpendicular to the line of intersection of the planes  $2x + y - z = 3$  and  $x + 2y + z = 2$ .
32. The acceleration of a particle at time  $t$  is given by  $a = (12 \cos 2t) i - (8 \sin 2t) j + 16t k$ . If the velocity  $v$  and the displacement  $r$  be zero at  $t = 0$ . Find
- (i)  $v$  and
- (ii)  $r$  at time  $t$ .
33. Find the radius of curvature of  $\frac{x^2}{9} + \frac{y^2}{16} = 2$  at  $(3, 2)$ .

(6 x 5 = 30 Marks)

#### Part D

Answer any Two questions. Each question carries 10 marks

34. a) Solve the polynomial equation  $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$  of which one root is  $-1 + \sqrt{-1}$
- b) Solve the cubic  $x^3 - 9x + 28 = 0$  by Cardano's method.
35. Verify that the matrix  $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$  satisfies its characteristic equation and hence find  $A^{-1}$ .
36. Find  $T, N, B, \kappa$  and  $\tau$  for the space curve  $r(t) = (3 \cos t) i + (3 \sin t) j + 2t k$ .

(2 x 10 = 20 Marks)

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(12 x 1 = 12 Marks)

#### Part E

Answer any Two questions. Each question carries 3 marks

37. Form the polynomial equation of fourth degree with rational coefficients, one of whose roots is  $\sqrt{2} + \sqrt{3}$ .