

15U307

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Name:

Reg. No.....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS - UG)

Mathematics - Complementary Course

CC15U MAT3 C03- MATHEMATICS

(2015 - Admission)

Time: Three Hours

Maximum: 80 Marks

PART A

Answer all questions (Each carries 1 mark)

1. Define Bernoulli equation
2. Check for exactness :
 $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$.
3. What is the order and degree of the differential equation $y'' - (y')^3 + 4 = 0$.
4. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
5. The roots of the equation $|A - \lambda I| = 0$ is called
6. The eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ are
7. Find $\text{curl } \vec{v}$ where $\vec{v} = [2y, 5x, 0]$.
8. $(i, j, k) = \dots\dots\dots$
9. $\text{div}(\text{curl } \vec{v}) = \dots\dots\dots$
10. Define Solenoidal vector.
11. Write the parametric representation of the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$.
12. State Gauss's Divergence theorem.

(12 x 1 = 12 marks)

PART B

Answer any NINE questions (Each carries 2 marks)

13. Find the orthogonal trajectories of the family of curves $y = ce^{-x^2}$.
14. Show that y^2 is an integrating factor of $y \cos x dx + 3 \sin x dy = 0$.
15. Solve the differential equation $(1 - x^2) \frac{dy}{dx} - xy = 1$.
16. Obtain the row equivalent canonical matrix C of $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 6 & 8 \end{bmatrix}$
17. Find the rank of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by reducing it to the normal form.
18. Find the eigen values of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$
19. Test for consistency of the equations: $2x + 3y = 13$
 $5x - 2y = 4$

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20. Find the angle between the planes $x + y + z = 1$ and $x + 2y + 3z = 6$
21. Find a unit vector perpendicular to the vectors $[2, 3, 4]$ and $[-2, 1, 1]$.
22. Find $\text{div } \vec{v}$ where $\vec{v} = xyz\hat{i} + 3zx\hat{j} + z\hat{k}$ at $(1, 2, 3)$.
23. Check whether the vectors $[2, -3, 4]$, $[1, 2, -1]$ and $[3, -1, 2]$ are linearly independent or not.
24. Find the directional derivative of the function $2xy + z^2$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$ at the point $(1, -1, 3)$. (9 x 2 = 18 marks)

PART C

Answer any **SIX** questions (Each carries 5 marks)

25. Solve the differential equation $2xy \frac{dy}{dx} - y^2 + x^2 = 0$.
26. Solve the equation $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$.
27. Solve the differential equation $(2x - 4y + 5)y' + (x - 2y + 3) = 0$.
28. Solve the following system of equations using Cramer's rule: $2x - y + 3z = 9$
 $x + y + z = 6$
 $x - y + z = 2$
29. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ and hence obtain A^{-1} .
30. Find the tangential and normal acceleration of a body moving along a path with position vector $\mathbf{r}(t) = e^t \hat{i} + e^{-t} \hat{j}$.
31. Find the workdone in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.
32. Using Green's theorem, evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
33. Using Divergence theorem, Evaluate $\iint_S (7x\hat{i} - z\hat{k}) \cdot \vec{n} dA$ over the sphere $x^2 + y^2 + z^2 = 4$ (6 x 5 = 30 marks)

PART D

Answer any **TWO** questions (Each carries 10 marks)

34. Solve the differential equation $x(1 - x^2) \frac{dy}{dx} + (2x^2 - 1)y = ax^3$.
35. Solve the system of equations : $x + y + 2z = 4$
 $2x - y + 3z = 9$
 $3x - y - z = 2$.
36. Verify Stokes theorem for $\vec{F} = [y, z, x]$ over the the paraboloid $z = 1 - (x^2 + y^2)$, $z \geq 0$. (2 x 10 = 20 marks)
