

15U213

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Name.....

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, JUNE 2016
(CUCBCSS – UG)

(Complementary Course: Mathematics)
CC15UMAT2C02 – MATHEMATICS

(2015 Admission)

Time: Three Hours

Maximum: 80 Marks

PART A

Answer All Questions. Each question carries 1 mark

1. The range of the function $y = \cosh x$ is
2. The derivative of $\sin^{-1}(\tanh x)$ is
3. Find $\int_0^1 \frac{1}{\sqrt{x}} dx$
4. For the sequence 0, 3, 8, 15, 24,, find the n^{th} term.
5. $\lim_{n \rightarrow \infty} \left(1 + \frac{7}{n}\right)^n = \dots\dots\dots$
6. $\lim_{n \rightarrow \infty} \sqrt[n]{3n} = \dots\dots\dots$
7. Find $\lim_{(x,y) \rightarrow (1,1)} \frac{3x^2 - y^2}{x^2 + y^2 + 2}$
8. What is the polar equation of the circle with centre $\left(-1, \frac{\pi}{2}\right)$ and radius 2?
9. If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \rightarrow \infty} a_n = \dots\dots\dots$
10. Convert the rectangular co-ordinates (0,1,0) into spherical co-ordinates.
11. Find the Cartesian form of the polar equation $r = \sin 2\theta$.
12. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.

(12 × 1 = 12 Marks)

PART B

Answer any NINE Questions. Each question carries 2 marks

13. Evaluate $\int \sinh^{-1} x dx$.
14. Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $y = 0$, $x = 2$ about the x-axis.
15. Examine the convergence of the integral $\int_2^{\infty} \frac{1}{\ln x} dx$.
16. Polar equation of a conic is $r = \frac{12}{3+3 \sin \theta}$. Identify the conic and find its directrix.
17. Find the derivative of $y = 2\sqrt{t} \tanh \sqrt{t}$ with respect to t.
18. Convert the rectangular co-ordinates (-1,1,2) into cylindrical co-ordinates.
19. Evaluate $\lim_{n \rightarrow \infty} \frac{n!}{n^n}$
20. Show that the series $\sum_1^{\infty} n^2$ diverges.
21. Define boundary point. Give an example.
22. Express the repeating decimal 5.232323..... as a ratio of two integers.
23. Show that the series $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$ converges absolutely for all values of x.

24. Find the Taylor series expansion of $f(x) = e^x$ at $x = 0$.

(9 × 2 = 18 Marks)

PART C

Answer any **SIX** Questions. Each question carries 5 marks

25. Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ for all real x .

26. Find the area of the surface generated by revolving the curve $x = 2\sqrt{4 - y}$, $0 \leq y \leq \frac{15}{4}$ about the y -axis.

27. Show that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

28. If $f(x, y) = x \cos y + ye^x$, find f_{xy} and f_{yx} .

29. Show that the following series converges to $\ln(1 + x)$ for all $-1 < x < 1$

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

30. If $0 < b < 1$, then show that $\lim_{n \rightarrow \infty} b^n = 0$.

31. Find the linearization $L(x, y, z)$ of the function $f(x, y, z) = e^x + \cos(y + z)$ at $(0, 0, 0)$.

32. Find the area shared by the circle $r = 2$ and the cardioid $r = 2(1 - \cos \theta)$.

33. Find the length of the cardioid $r = a(1 + \cos \theta)$.

(6 × 5 = 30 Marks)

PART D

Answer any **TWO** Questions. Each question carries 10 marks

34. (a) Find the spherical co-ordinate equation for the sphere $(x - 2)^2 + (y - 1)^2 + z^2 = 16$.

(b) If $f(u, v, w)$ is differentiable and $u = x - y$, $v = y - z$ and $w = z - x$, then show that

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0.$$

35. Use partial fractions to find the sum of the series $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$

36. Define the interval and radius of convergence of a power series. Find the interval in which the following series converges

$$1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \frac{x^4}{5} - \dots$$

(2 × 10 = 20 Marks)
