15U228

(Pages: 3) Name.....................

Reg. No.................

SECOND SEMESTER B. Sc DEGREE EXAMINATION, JUNE 2016

(CUCBCSS – UG)

(Complementary Course: Statistics)

CC15U ST2 C02-Probability Distributions

(2015 Admission)

Time: Three Hours Maximum: 80 Marks

Section A

*(One word questions. Answer **all** questions. Each question carries 1 mark)*

Fill up the blanks:

1. E|X-A| is minimum when A is ....
2. If Var (X) = 1, then Var (2X ± 3) = ...
3. Poisson distribution is a limiting case of binomial distribution under the conditions ....
4. The points of inflexion for a normal curve are ....
5. The p.d.f of Gamma distribution is ....

Write true or false

6. Expected value of a random variable always exists.
7. Mean of Poisson distribution is 2 and variance is 5
8. If X and Y are two independent normal variates, then X-Y is also a normal variate.
9. The mode of a binomial distribution having mean 6 and variance 2 is 6.
10. Central limit theorem can be applied only for Poisson distribution.

(10×1=10 marks)

Section B

(*One sentence questions. Answer **all** questions. Equestion carries 2 marks*)

11. Define mathematical expectation.
12. Define characteristic function of a random variable.
13. X and Y are independent random variables with means 10 and 20 and variances 2 and 3 respectively. Find the variance of 3X+ 4Y.
14. Define conditional variance.

15. Define joint probability density function in discrete and continuous cases.
16. Find the m. g. f. of a random variable for which $\mu_r' = r!$
17. Define convergence in probability.

(7×2=14 marks)

Section C

*(Paragraph questions. Answer any **three** questions. Each question carries **4** marks)*

18. If X is a random variable and a and b are constants, then show that $E(aX+b) = a\,E(X) + b$
19. State and prove addition theorem of expectation.
20. A two dimensional random variable (X, Y) has the joint density
$$f(x, y) = \begin{cases} kx^2y \\ 0 \; elsewhere \end{cases}, \; 0 < x, y < 1$$
find (1) the constant k (ii) $P\{0 < X < \frac{3}{4}, \frac{1}{2} < Y < 1\}$
21. State and prove the reproductive property of the Poisson distribution.
22. X is normally distributed with mean 12 and standard deviation 4. Find the probability that (i) $0 \le X \le 12$; (ii) $X \ge 20$.

(3 × 4 = 12marks)

Section D

*(Short Essay questions. Answer any **four** questions. Each question carries **6** marks)*

23. Find the mean and variance if the distribution function of the random variable X is
$$F(x) = \begin{cases} 0 \; if \; x < -1 \\ \frac{1+x}{2} \; if -1 \le x < 1 \\ 1 \; if \; x \ge 1 \end{cases}$$
24. Define central and raw moments. Derive an expression for the r^{th} central moment in terms of the raw moments.
25. Two random variables X and Y have the joint density :
$$f(x, y) = \begin{cases} 8xy, 0 < x < y < 1 \\ 0 \quad elsewhere \end{cases}.$$
Find (i) E(Y/X = x) , (ii) Var (Y/X = x) .
26. If X is a random variable with a continuous distribution function F, then prove that F(X) has a uniform distribution on [0 , 1].
27. Derive the recurrence relation for the central moments of binomial distribution.
28. State and prove the Bernoulli's law of large numbers.

(4×6=24 marks)

Section E

*(Essay questions. Answer any **two** questions. Each question carries **10** marks)*

29. Let X and Y have the joint probability mass function given by

$$p(x, y) = \begin{cases} \frac{3}{16} \; for\ (x,y) = (-1,0), (-1,1), \; (1,0), \; (2,1) \\ \frac{1}{16} \; for\ (x,y) = (1,2), \; (0,3), \; (-1, \; 2), (1,3) \end{cases}$$

Find the correlation coefficient between X and Y.

30. The random variables X and Y have the joint distribution given by the p.d.f. :

$$f(x, y) = \begin{cases} 6(1-x-y), \; for\ x > 0, y > 0, x + y < 1 \\ 0 \; else\ where. \end{cases}$$

Find the marginal distributions of X and Y. Hence examine if X and Y are independent.

31. The mean I.Q (intelligence quotient) of a large number of children of age 14 was 100 and the standard deviation 16. Assuming that the distribution of I.Q was normal, find
    (i) What percentage of the children had I.Q. under 80?
    (ii) Between what limits the I.Q's of the middle 40% of the children lay?
    (iii) What percentage of the children had I.Q's within the range $\mu \pm 1.96$?

32. State and prove Chebyshev's inequality.

(2× 10=20 marks)

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