**16U407** Pages: 2 Name : ..............................

Reg. No............................

**FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2018**

(Regular/Supplementary/Improvement)

(CUCBCSS-UG)

**CC15U MAT4 C04 - MATHEMATICS**

(Mathematics - Complementary Course)

(2015 Admission onwards)

Time : Three Hours Maximum : 80 Marks

## Part A

Answer ***all*** questions. Each question carries 1 mark.

1. Define non-homogeneous linear differential equations of second order.
2. What are the characteristic roots of the equation $4y'' + 4y' - 3y = 0$.
3. State Existance and Uniqueness theorem for initial value problems.
4. Find $L(e^{at} \sinh at)$.
5. Show that Laplace transform is a Linear operation.
6. Find $L^{-1}\left(\frac{1}{s^2+\omega^2}\right)$.
7. Give example of an even function.
8. If $f(x)$ and $g(x)$ are odd functions then $f(x)g(x)$ is again an odd function. State True or false. Justify your answer.
9. Write the degree of $x\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0$.
10. Write the three dimensional Laplace Equation.
11. Solve $u_{xx} + 4u = 0$.
12. Using Trapezoidal rule, Evaluate $\int_a^b f(x)dx$.

**(12 x 1 = 12 Marks)**

## Part B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Find the general solution of $(x^2D^2 + 1.25)y = 0$.
14. Distinguish between general and particular solution.
15. Verify that a general solution of $y'' - 2y' + y = 0$ on any interval is $y = (c_1 + c_2x)e^x$.
16. State and prove First shifting theorem for Laplace transforms.
17. Find $L\left(e^{-3t}[2\cos 5t - 3\sin 5t]\right)$.
18. Using convolution, find $L^{-1}\left(\frac{1}{(s^2+1)^2}\right)$.
19. Define fundamental period and find the fundamental period of $\tan 4x$.

20. Find a Fourier series to represent $f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } \ 0 < x < \pi \end{cases}$
where $f(x+\pi) = f(x)$.

21. Verify Laplace equation for $u = \ln(x^2 + y^2)$.

22. Using Picard's method find an approximate solution to $y' = 1 + y^2, y(0) = 0$.

23. Evaluate $\int_0^2 \frac{1}{x+1}\, dx$ with $n = 4$ using Simpson's rule.

24. Evaluate $\int_0^\pi \sin x dx$ using Trapezoidal rule with $n = 6$.

**(9 x 2 = 18 Marks)**

## Part C

Answer any ***six*** questions. Each question carries 5 marks.

25. Using the method of reduction of order solve $x^2y'' - 5xy + 9y = 0$, given that $y = x^3$ is a solution.

26. Solve $y'' - 6y' + 13y = 4e^{3x}$, $y(0) = 2$, $y'(0) = 4$.

27. Find the general solution of the equation $4y'' + y = \csc x$.

28. Find the Laplace transform of $t^2 \sin \omega t$

29. Find the Laplace transform of the function

$$f(t) = \begin{cases} 2 & \text{if } \ 0 < t < \pi \\ 0 & \text{if } \ \pi < t < 2\pi \\ \sin t & \text{if } \ t > 2\pi \end{cases}$$

30. Find $L^{-1}\left(\frac{5s^2 - 15s - 11}{(s+1)(s-2)^3}\right)$.

31. Find the Fourier series representing $x$ in the interval $[-\pi, \pi]$. Deduce that $1 - \frac{1}{3} + \frac{1}{5} - ... = \frac{\pi}{4}$

32. Solve $u_{xy} = u_x$.

33. Find an upperbound for the error incurred in estimating the integral $\int_0^\pi x \sin x dx$ using trapizoidal rule with $n = 10$.

**(6 x 5 = 30 Marks)**

## Part D

Answer any ***two*** questions. Each question carries 10 marks.

34. Solve $y'' - 3y' + 2y = 4e^{2t}$, $y(0) = -3$, $y'(0) = 5$ using Laplace transform.

35. Obtain the Fourier series for the function

$$f(x) = \begin{cases} x - \pi & \text{if } \ -\pi < x < 0 \\ \pi - x & \text{if } \ 0 < x < \pi \end{cases}$$

Use it to find a series for $\pi^2$.

36. Apply Runge-Kutta method to solve the initial value problem $y' = x+y$, $y(0) = 0$ choosing $h = 0.2$ and computing $y_1, y_2, y_3$. Compare then with the actual values.

**(2 x 10 = 20 Marks)**

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