C 80028 (Pages : 3) Name..............................

Reg. No..............................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2015

(U.G.–CCSS)

Core Course—Mathematics

MM 6B 12—NUMBER THEORY AND LINEAR ALGEBRA

Time : Three Hours Maximum : 30 Weightage

Section A

Answer all **twelve** *questions.*

1. Find g.c.d. (143, 227).
2. State fundamental theorem of Arithmetic.
3. Express 4725 in canonical form.
4. State Feromat's little theorem.
5. Find the sum of divisions of 180.
6. When will you say a number theoretic function f is multiplicative.
7. Find ϕ (360).
8. Define rank of a matrix.
9. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$.
10. Find characteristic root of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.
11. State the nature of the characteristic roots of Hermitian matrices.
12. State Cayley Hamilton theorem.

($12 \times 1/4 = 3$ weightage)

Section B

Answer all **nine** *questions.*

13. Prove if g.c.d. $(a, b) = d$ then g.c.d. $\left(a/d, b/d\right) = 1$.
14. Use of Euclidean algorithm to find x and y which satisfies g.c.d. $(56, 72) = 56x + 72y$.

Turn over

15. Check whether the following Diophantine equation can be solved $6x + 51y = 22$.

16. Show $a^7 \equiv a \pmod{42}$ for all a.

17. Determine the highest power of 3 dividing 80!

18. Reduce to the normal form to find rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$.

19. State the Sylvester's law of nullity.

20. Show that the characteristic roots of a triangular matrix are just the diagonal elements of the matrix.

21. Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.

($9 \times 1 = 9$ weightage)

Section C

Anwer any **five** *questions.*

22. Prove there is an infinite number of primes.

23. If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then prove $a \equiv c \pmod{n}$.

24. Show $181 + 1 \equiv 0 \pmod{437}$.

25. Show $\sigma(n) = \sigma(n+1)$ if $n = 14$ where $\sigma(n)$ = sum of divisors of n.

26. Prove Euler's theorem, $a^{\phi(n)} \equiv 1 \pmod{n}$ if $n \geq 1$ and g.c.d. $(a, n) = 1$.

27. Find non-singular matrices P and Q such that PAQ is in the normal form where $A = \begin{bmatrix} 1 & 1 & \\ 1 & 2 & \\ 0 & -1 & \end{bmatrix}$

28. Solve the system of equations :

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0.$$

($5 \times 2 = 10$ weightage)

Section D

Anwer any **two** *questions.*

29 (a) Prove the fundamental theorem of arithmetic.

(b) Solve the linear congruence equation $6x \equiv 15 \pmod{21}$.

30 (a) State and prove Wilson's theorem.

(b) If n is an odd integer then prove $\phi(2n) = \phi(n)$.

31 Show that the equations :

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

$$x - y + z = -1.$$

are consistent and solve the same.

(2 × 4 = 8 weightage)