33

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2015

(U.G.—CCSS)

Core Course—Mathematics

MM 6B 09—REAL ANALYSIS

Time : Three Hours   Maximum : 30 Weightage

Part A

Answer **all** *questions.*

1. State Maximum Minimum theorem.

2. Give an example of a uniform continuous function.

3. Define norm of a partition.

4. When is a function $f : [a, b] \to \mathbb{R}$ is Riemann integrable ?

5. Define primitive of f.

6. If $J = J = [c, d] \subset [a, b]$ and $\varphi_j(x) = \begin{cases} 1, x \in J \\ 0, x \in [a, b] - J \end{cases}$. Find $\int_a^b \varphi_j$?

7. Show that $\dfrac{x}{n} = 0, \forall\, x \in \mathbb{R}$.

8. Evaluate $\lim \dfrac{nx}{1 + (nx)^2}$.

9. Find the radius of convergence of $\sum x^n$.

10. Define improper integral.

11. State True or false. "If f is continuous on $[1, \infty)$ and if $\int_1^\infty f(x)\, dx$ converges then $\lim\limits_{x \to \infty} f(x) = 0$.

12. What is $B(m, n)$?

$(12 \times \tfrac{1}{4} = 3 \text{ weightage})$

Turn over

Part B

*Answer **all** questions.*

13. Define Lipschitz function.

14. State an prove Bolzano's intermediate value theorem.

15. Show that every constant function on $[a, b]$ is $R[a, b]$.

16. Using First form of fundamental theorem prove that $\int_a^b x\,dx = \frac{1}{2}\left(b^2 - a^2\right)$.

17. If $f_n(x) = x^n, x \in [0, 1]$, check whether $f_n(x)$ converges to zero uniformly.

18. If (f_n) is a sequence of continuous functions converging uniformly to f, prove that f is contin

19. Show that $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ diverges.

20. Define Gamma function.

21. Show that $B(1, n) = 1/n$.

(9 × 1 = 9 weigh

Part C

*Answer any **five** questions.*

22. If f and g are uniform continuous on a subset A of R, then prove that $f + g$ is uniformly conti
    on A.

23. Show that $f(x) = xe^x - 2$ has a root in $[0, 1]$.

24. If $f \in R[a, b]$ then prove that f is bounded on $[a, b]$.

25. State and prove Weierstrass M test for the series.

26. If $\int_a^\infty |f(x)|\,dx$ converges absolutely then prove that $\int_a^\infty f(x)\,dx$ converges.

27. Show that $\int_0^1 \dfrac{1}{x}\,dx$ diverges.

28. Prove the recurrence formula for gamma function.

(5 × 2 = 10 weightage)

Part D

*Answer any **two** questions.*

29. State and prove Squeeze theorem. Also prove that a step function $\varphi : [a, b] \to \mathrm{R}$ is in $\mathrm{R}[a, b]$.

30. State and prove Cauchy's criterion for uniform convergence.

31. Show that $\int_0^1 \dfrac{1}{\sqrt{1 - x^2}}\,dx$ and $\int_0^\infty \dfrac{1}{x^2 + \sqrt{x}}\,dx$ are convergent.

(2 × 4 = 8 weightage)