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SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2015

(U.G.–CCSS)

Core Course—Mathematics

MM 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 30 Weightage

Answer **all** *questions.*

1. Define a harmonic function.
2. Sin $(i\,y) =$ —————.
3. Given $f(z) = \dfrac{z^3 (z-1)^4 (z+1)^5}{(z+2)^2 (z+6)}$. Write the order of the zero $z = 1$.
4. State Cauchy's residue theorem.
5. $\operatorname{Cos} h^2 z - \sin h^2 z =$ —————.
6. Define isolated singularity.
7. If $e^z = e^{x+iy}$ then $\arg(e^z) =$ —————.
8. $\int_c \dfrac{z^2}{z-3}\, dz =$ ————— where c is the circle $|z| = 2$.
9. Prove that $u = x^2 - y^2$ is harmonic.
10. State Liouville's theorem.
11. Verify Cauchy-Riemann equation for the function $f(z) = (3x + y) = i\,(3y - x)$.
12. Define Pole.

(12 × ¼ = 3 weightage)

Answer all **nine** *questions.*

13. Find the harmonic conjugate of $u = x^4 - 6x^2y^2 + y^4$.
14. Prove that $f'(z)$ does not exist at any point if $f(z) = z - \bar{z}$.

Turn over

15. Prove that $f(z) = \frac{\bar{z}}{z}$ does not have a limit when $z \to 0$.

16. Evaluate $\int_c \frac{e^z}{z^5}\, dz$ where c is $|z| = 1$.

17. Give an example of removable singularity.

18. Evaluate the residue at the pole $z = 1$ of $f(z) = \frac{z+1}{z^2(z-1)}$.

19. Determine the order of zero of the function $z\left(e^z - 1\right)$ at $z = 0$.

20. Find the principal value of $(-i)^i$.

21. Prove that $\exp\left(\frac{2+\pi i}{4}\right) = \sqrt{e}\left(\frac{1+i}{\sqrt{2}}\right)$.

(9 × 1 = 9 weigh

Answer any **five** *questions.*

22. Find an analytic function $f(z) = u + iv$ given $u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$.

23. Prove that $\text{Log}(1-i) = \frac{1}{2}\text{Log } 2 - i\frac{\pi}{4}$.

24. State and prove Cauchy's Integral formula.

25. Using Taylor's series prove that

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}}.$$

26. Using Cauchy's residue theorem evaluate :

$$\int_c \frac{z+1}{z^2}\, dz \text{ where c is } |z| = 1.$$

27. Prove that differentiable functions are continuous.

28. Let $f(z)$ be an analytic function such that $|f(z)| \leq A|z|$ for every z, where A is constant. Prove that $f(z) = a, z$ where a, is a complex constant.

(5 × 2 = 10 weightage)

Answer any **two** *questions.*

29. Obtain Laurent series expansion of $\frac{1}{(z-1)(z-2)}$ in $| < |z| < 2$.

30. Prove that $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} & : z \neq 0 \\ 0 & : z = 0 \end{cases}$ is not analytic at $z = 0$. But Cauchy-Riemann equations are satisfied at that point.

31. Evaluate $\int_0^\infty \frac{1}{(x^2+1)^2}\, dx$.

(2 × 4 = 8 weightage)