Setting target values for inefficient DMUs in Integer DEA Models

K G Shinto[1] and C M Sushama[2]
[1](Department of Mathematics, Christ College (Autonomous) Irinjalakuda, India, shintomaths@gmail.com)
[2](Department of Mathematics, National Institute of Technology Calicut, India)

Abstract: *Conventional DEA model assumes real valued inputs and outputs and measures the relative efficiency of similar decision making units (DMUs). If a particular DMU is not efficient, a set of target values for the input/output can be suggested by projecting the DMU to the efficient frontier which will make the DMU efficient. But in a DEA model with integer restriction in input and/or output values, these target values may not be acceptable if they are not integers. Rounding off these values may not result in an integer efficient point in all cases. This paper presents an iterative method to determine integer target values for inputs and/or outputs in the case of inefficient DMUs so as to function efficiently. Different algorithms are explained to handle input oriented, output oriented and additive integer DEA models. An example from literature is considered to compare these models.*

Keywords: *Data Envelopment Analysis (DEA), Mixed integer linear programming (MILP), CRS DEA, VRS DEA, Additive DEA*

I. INTRODUCTION

Data envelopment analysis (DEA) is a mathematical programming approach used to measure the relative efficiency of decision making units (DMUs) that converts homogeneous multiple inputs into homogeneous multiple outputs and classify the set into two categories, efficient and inefficient. It is also used to set target values for inefficient DMUs to perform efficiently. Traditional models assume real valued inputs and outputs. But there are many occasions in which some inputs and/or outputs can assume only integer values. For example, number of buses, number of doctors, number of passengers, and number of patients etc. as inputs or outputs. In such situations target setting by conventional models may not provide the acceptable integer values for inputs and outputs. However the rounding off the targets to the nearest integer can be used for situations with large inputs and outputs. But this will not work in all situations. Lozano and Villa [11] addressed the shortcomings of the conventional DEA models in rounding up/down for the target values. They proposed an integer production possibility set and an integer efficient frontier with an MILP DEA model to guarantee the required integrality by projecting an inefficient DMU into the integer efficient frontier. This is the first research work to handle the integer restriction in the DEA models. Soon after this work, Kuosmanen and Kazemi Matin [10] pointed out the theoretical inadequacy in the model and the efficiency of the DMU after setting integer targets. They also proposed an MILP model for the same. However Khezrimotlagh et al [8] claimed that the mentioned drawbacks of the Lozano’s model is not valid and they pointed out some drawbacks for Kuosmanen and Kazemi Matin’s model [10] and suggested their own models[8,9].

In all the existing integer DEA models, the target input, output values are obtained by some specified method such as Lozano and Villa [8],Kuosmanen and Kazemi Matin [10], Khezrimotlagh [9] etc. But these target values of different models are different and in almost all models, they claimed that their method is more efficient than the others. The target values suggest by these models are obtained by the projection of an inefficient DMU into one of the integer efficient point. If this point coincides with one of the CCR efficient DMU then it will have the CCR efficiency score $\boldsymbol{\theta = 1}$and hence become the most effective DMU. If a particular DMU is projected into a CCR efficient DMU then it will attain the maximum efficiency. So to attain maximum efficiency one need only to project the inefficient DMU to one of the CCR efficient DMU. This paper introduces atwo phase iterative MILP algorithm to tackle the situation by suggesting integer target values in the integer efficient frontier.

The rest of the paper is organised as follows. In section 2, the basic definitions and theoretical background of the proposed algorithm is explained. In section 3 the algorithm for CRS input oriented integer model is explained. In section 4, the interpretation of the algorithm is considered. In section 5 the computational algorithm for output oriented and additive integer CRS DEA models and input oriented VRS model is given. In Section 6 the extension of the algorithm for target setting is considered. Applicability of the algorithm is illustrated in section 7 using a data set from the literature.

II. THEORETICAL BACKGROUND

Let $\boldsymbol{X} = (\overline{\boldsymbol{X}}_1, \overline{\boldsymbol{X}}_1 \dots \dots \overline{\boldsymbol{X}}_n)$and $\boldsymbol{Y} = (\overline{\boldsymbol{Y}}_1, \overline{\boldsymbol{Y}}_1 \dots \dots \overline{\boldsymbol{Y}}_n)$ be the input and output matrices where $\overline{\boldsymbol{X}}_j$ and $\overline{\boldsymbol{Y}}_j$ denote the input and output vectors for DMU$_j$ for $\boldsymbol{j} = \mathbf{1}, \mathbf{2}, \dots \dots . \boldsymbol{n}$. The production possibility set corresponding to the CRS technology is given by

$$\boldsymbol{T_{CRS}} = \{(\overline{\boldsymbol{X}}, \overline{\boldsymbol{Y}}) : \exists \lambda_1, \lambda_1, \dots . . \lambda_n ; \lambda_j \geq \mathbf{0} \forall \boldsymbol{j} \quad \overline{\boldsymbol{X}} \geq \boldsymbol{X}\boldsymbol{\lambda}, \overline{\boldsymbol{Y}} \leq \boldsymbol{Y}\boldsymbol{\lambda}\}$$

The efficient frontier is a subset of the PPS formed by the set of all non dominating points. That is,

$$\boldsymbol{T_{CRS}^{eff}} == \{(\overline{\boldsymbol{X}}, \overline{\boldsymbol{Y}}) \in \boldsymbol{T_{CRS}} : \forall (\overline{\boldsymbol{X}}', \overline{\boldsymbol{Y}}') \in \boldsymbol{T_{CRS}}, (\overline{\boldsymbol{X}}' \leq \overline{\boldsymbol{X}}) \cap (\overline{\boldsymbol{Y}}' \geq \overline{\boldsymbol{Y}}) \leftrightarrow (\overline{\boldsymbol{X}}', \overline{\boldsymbol{Y}}') = (\overline{\boldsymbol{X}}, \overline{\boldsymbol{Y}})\}$$

Now if some or all of the input and/or output components can take only integer values, then the PPS of such DEA models is the subset of $\boldsymbol{T_{CRS}}$ satisfying the integer restriction. Let $\boldsymbol{I} = \{\mathbf{1}, \mathbf{2}, \dots . \boldsymbol{m}\}$ and $\boldsymbol{O} = \{\mathbf{1}, \mathbf{2}, \dots \dots \boldsymbol{p}\}$ be the set of input and output dimensions and let $\boldsymbol{O}' \subseteq \boldsymbol{O}$ and $\boldsymbol{I}' \subseteq \boldsymbol{I}$ be the dimensions for integer valued. Then the CRS integer PPS is given by

$$\begin{aligned}\boldsymbol{T}'_{\boldsymbol{CRS}} = \{(\overline{\boldsymbol{X}}, \overline{\boldsymbol{Y}}) &: \exists \lambda_1, \lambda_1, \dots . . \lambda_n ; \lambda_j \geq \mathbf{0} \forall \boldsymbol{j} \\ &\overline{\boldsymbol{X}} \geq \boldsymbol{X}\boldsymbol{\lambda}, \overline{\boldsymbol{Y}} \leq \boldsymbol{Y}\boldsymbol{\lambda}; \overline{\boldsymbol{X}}_i \in \boldsymbol{Z}, \forall \boldsymbol{i} \in \boldsymbol{I}'; \\ &\overline{\boldsymbol{Y}}_i \in \boldsymbol{Z}, \forall \boldsymbol{i} \in \boldsymbol{O}'\}\end{aligned}$$

DMU$_j$ is said to be integer efficient if no other integer valued point of PPS dominates it. The CRS integer efficient frontier is therefore the set of non dominated integer valued points of the PPS.

Now if a particular DMU is CRS efficient then it will be in the efficient frontier $\boldsymbol{T_{CRS}^{eff}}$. Again since its target values are the given values, it satisfies the integer restriction and so it will be in the integer efficient frontier. That is, if DMU$_j$ is CRS efficient, then it is CRS integer efficient. The converse is obviously not true.

The classical input oriented CRS model is defined as follows

$$\boldsymbol{min}\, \boldsymbol{\theta_j} - \epsilon(\boldsymbol{S}^- + \boldsymbol{S}^+)$$

Subject to $\boldsymbol{X}\boldsymbol{\lambda} - \boldsymbol{\theta}\overline{\boldsymbol{X}}_j + \boldsymbol{S}^- = \mathbf{0}$

$$\boldsymbol{Y}\boldsymbol{\lambda} - \boldsymbol{S}^+ = \overline{\boldsymbol{Y}}_j$$

$$\boldsymbol{\lambda}, \boldsymbol{S}^-, \boldsymbol{S}^+ \geq \mathbf{0}$$

It is a two phase LPP algorithm. In phase I maximum $\boldsymbol{S}^- + \boldsymbol{S}^+$ is considered and using this minimum $\boldsymbol{\theta}$ is considered in phase II. DMU$_j$ is CRS efficient if $\boldsymbol{S}^- = \boldsymbol{S}^+ = \mathbf{0}$ and$\boldsymbol{\theta_j} = \mathbf{1}$. If DMU$_j$ is not CRS efficient, then either $\boldsymbol{\theta_j} < 1$ or $\boldsymbol{S}^-\, \boldsymbol{or}\, \boldsymbol{S}^+$ is non zero. The target values for such a DMU is given by

$$\widehat{\boldsymbol{X}}_j = \boldsymbol{\theta_j}\overline{\boldsymbol{X}}_j - \boldsymbol{S}^-$$

$$\widehat{\boldsymbol{Y}}_j = \overline{\boldsymbol{Y}}_j + \boldsymbol{S}^+$$

to make the DMU efficient. With this transformation the DMU will project into the efficient frontier and will thus become efficient.

Now if there is integer restriction then the target $(\widehat{\boldsymbol{X}}, \widehat{\boldsymbol{Y}})$ should be integer vectors. Lozano and Villa [11] introduced a model to solve CRS integer valued problems. It is a two phase model and is as follows

$$\boldsymbol{min}\, \boldsymbol{\theta_j} - \epsilon\left(\sum_i s_i^- + \sum_k s_k^+\right)$$

subject to

$$\sum_j \lambda_j x_{ij} = x_i \forall \boldsymbol{i}$$

$$x_i = \theta_j x_{ij} - s_i^-$$

$$\sum_j \lambda_j\, y_{kj} = y_k \forall \boldsymbol{k}$$

$$y_k = y_{kj} + s_k^+$$

$$\lambda_j, s_i^-, s_k^+, x_i, y_k \geq \mathbf{0}$$

x_i, y_k are integers $\forall \boldsymbol{i} \in \boldsymbol{I}' \& \forall k \in O'$

Where $\boldsymbol{\theta_j}$ is the CRS integer efficiency score. An existing DMU, DMU$_j$ is CRS integer efficient if and only if$\boldsymbol{\theta_j} = \mathbf{1}, s_i^- = \mathbf{0}\ \forall \boldsymbol{i}\ \& s_k^+ = \mathbf{0}\ \forall \boldsymbol{k}$. Since for an efficient DMU $\boldsymbol{\theta_j} = \mathbf{1}$, it is desirable to consider a model with $\boldsymbol{\theta_j} = \mathbf{1}$.

Proposition 1

In the Lozano and Villa model if $\boldsymbol{\theta_j} = \mathbf{1}$, then the slacks s_i^- and s_k^+ should be integers for all $\boldsymbol{i} \in \boldsymbol{I}'$ and for all $\boldsymbol{k} \in \boldsymbol{O}'$

So with the help of this result it is possible to design a model as an MILP by a modification to the basic CCR model such that $\boldsymbol{\theta_j} = \mathbf{1}$ and $\boldsymbol{S}^-, \boldsymbol{S}^+$ as integer vectors. In this model if $\boldsymbol{S}^- = \boldsymbol{S}^+ = \mathbf{0}$, then the corresponding DMU should be integer efficient. If either $\boldsymbol{S}^-$ or $\boldsymbol{S}^+$ is non zero, then the corresponding DMU is not efficient and to achieve efficiency it is required to revise the input, output vectors of the DMU. For this the transformations

$\hat{X}_j = \bar{X}_j - S^-$

$\hat{Y}_j = \hat{Y}_j + S^+$

can be used. The following result ensures the applicability of the proposed algorithm.

Proposition 2

Let $(\lambda^*, S^{-*}, S^{+*})$ be the optimal solution of the MILP

$max\ S^- + S^+$

Subject to

$X\lambda + S^- = \bar{X}_j$

$Y\lambda - S^+ = \bar{Y}_j$

$\lambda, S^-, S^+ \geq 0$

S^-, S^+ are integers

Where X and Y are the matrices given by $X = (\bar{X}_1, \bar{X}_2 \ldots \bar{X}_n)$ and $Y = (\bar{Y}_1, \bar{Y}_2 \ldots \bar{Y}_n)$. Then the optimal solution $(\hat{\lambda}, \widehat{S^-}, \widehat{S^+})$ of the MILP

$max\ S^- + S^+$

Subject to

$X^*\lambda + S^- = \bar{X}_j^*$

$Y^*\lambda - S^+ = \bar{Y}_j^*$

$\lambda, S^-, S^+ \geq 0$

S^-, S^+ are integers

has the property that $\widehat{S^-} = \mathbf{0}$ and $\widehat{S^+} = \mathbf{0}$, where $\bar{X}_j^*$ is the column vector given by the transformation $\bar{X}_j^* = \bar{X}_j - S^{-*}$ and $\bar{Y}_j^*$ is the column vector given by the transformation $\bar{Y}_j^* = \bar{Y}_j + S^{+*}$ and X^* and Y^* are the matrices obtained by replacing the j[th] columns $\bar{X}_j$ by $\bar{X}_j^*$ and $\bar{Y}_j$ by $\bar{Y}_j^*$ respectively.

The proof of the theorem is obvious because in the optimal solution of the given MILP S^{-*} and S^{+*} has maximum possible integer values. So with the transformations $\bar{X}_j^* = \bar{X}_j - S^{-*}$ and $\bar{Y}_j^* = \bar{Y}_j + S^{+*}$, these maximum values will reduce to zero. Hence in the optimal solution of the second MILP $\widehat{S^-} = \mathbf{0}$ and $\widehat{S^+} = \mathbf{0}$.

III. COMPUTATIONAL ALGORITHM

The computational algorithm for the proposed input oriented CCR integer model is as follows:

Step 1: Measure the efficiency of the DMU_j using the classical input oriented CCR CRS model. If it is CRS efficient, then the integer target for DMU_j is $(\bar{X}_j, \bar{Y}_j)$ and stop.

Otherwise go to step 2

Step 2: Solve the MILP

$max\ S^-$

Subject to

$X\lambda + S^- = \bar{X}_j$

$Y\lambda - S^+ = \bar{Y}_j$

$\lambda, S^-, S^+ \geq 0$

S^-, S^+ are integers

If $S^- \neq \mathbf{0}$, then set $\bar{X}_j = \bar{X}_j - S^-$ and go to step 2

Otherwise set $\bar{Y}_j = \bar{Y}_j + S^+$

Where S^- and S^+ are taken from the optimum solution of the MILP of step 2.

Then go to step 3.

Step 3: Solve the transformed MILP with new objective function $max\ S^- + S^+$.

If either $S^- \neq \mathbf{0}$ or $S^+ \neq \mathbf{0}$,then set

$\bar{X}_j = \bar{X}_j - S^-$

$\bar{Y}_j = \bar{Y}_j + S^+$

$(\bar{X}_j, \bar{Y}_j)$ is the target value satisfying the integer restrictions. The integer restriction for S^- and S^+ required only for those components whose respective input or output is integral.

IV. INTERPRETATION OF THE ALGORITHM

The algorithm starts with the conventional DEA. It is also possible to start the algorithm from the MILP of step 2. But it does not have any efficiency measure. So starting with the basic model helps to measure the efficiency of the given DMU with given input-output.

If the integer target is suggested from step 1, then the clearly the suggested target is the given input and the output of a CCR efficient DMU satisfying the integer restriction. So clearly it is on the integer efficient frontier.

Now if the integer target is suggested from step 3, then corresponding to this $(\bar{X}_J, \bar{Y}_J)$, $S^- = S^+ = 0$. Also this solution satisfies the efficiency value $\theta_J = 1$ for the Lozano and Villa model [ ] because this assumption was made initially to develop the algorithm. So in this case also the integer target lies on the integer efficient frontier.
As a consequence we get the following theorem and it ensures the efficiency of the algorithm.

Theorem

The target value obtained using the algorithm $(\bar{X}_J, \bar{Y}_J)$ is a point in the integer efficient frontier.
The proposed algorithm reduces the input excess of the considered DMU. It also reduces the output shortfall corresponding to the revised input. Again the algorithm maintains input excesses as well as the output shortfalls in the considered DMU to zero in step 3 using the objective function $max\ S^- + S^+$. So the algorithm proceeds to reduce the input of an inefficient DMU by keeping the given output. While obtaining the target input, if there is any output shortfall, then the algorithm adjusts it so that with the target input, output the DMU will place onto the integer efficient frontier.

The convergence of the algorithm is obvious. The computational algorithm uses the iterative procedure only in step 2. The iterative algorithm proceeds to reduce the input excess present in the considered DMU. The input excess should be an integer vector less than $\bar{X}_J$. So the convergence of the algorithm is very rapid. In most of the problems the algorithm terminates in the first iteration itself.

V. COMPUTATIONAL ALGORITHM FOR OTHER INTEGER DEA MODELS

Algorithm for output oriented CCR integer DEA model
Step 1: Measure the efficiency of the DMU_j using the classical output oriented CCR CRS model. If it is CRS efficient, then the integer target for DMU_j is $(\bar{X}_J, \bar{Y}_J)$ and stop.
Otherwise go to step 2
Step 2: Solve the MILP
$max\ S^+$
Subject to
$X\lambda + S^- = \bar{X}_J$
$Y\lambda - S^+ = \bar{Y}_J$
$\lambda, S^-, S^+ \geq 0$
S^-, S^+ are integers
If $S^+ \neq 0$, then set $\bar{Y}_J = \bar{Y}_J + S^+$ and go to step 2.
Otherwiseset$\bar{X}_J = \bar{X}_J - S^-$

Where S^- and S^+ are taken from the optimum solution of the MILP of step 2.
Then go to step 3.
Step 3: Solve the transformed MILP with new objective function $max\ S^- + S^+$.
If either $S^- \neq 0$ or $S^+ \neq 0$, then set
$\bar{X}_J = \bar{X}_J - S^-$
$\bar{Y}_J = \bar{Y}_J + S^+$
$(\bar{X}_J, \bar{Y}_J)$ is the target value satisfying the integer restrictions.

As in the case of input oriented, the target value obtained using the algorithm $(\bar{X}_J, \bar{Y}_J)$is a point in the integer efficient frontier. Also the algorithm reduces the output shortfall of the considered DMU to zero and if there is any positive input excess present when the output shortfall is zero, then the algorithm maintains it as zero. So in this case the algorithm proceeds to increase the output of the inefficient DMU so as to place it in the efficient frontier.

Algorithm for the additive CRS integer DEA model

Step 1: Measure the efficiency of the DMU_j using the classical CCR additive model. If it is CRS efficient, then the integer target for DMU_j is $(\bar{X}_J, \bar{Y}_J)$ and stop.
Otherwise go to step 2
Step 2: Solve the MILP
$max\ S^- + S^+$

Subject to
$X\lambda + S^- = \bar{X}_j$
$Y\lambda - S^+ = \bar{Y}_j$
$\lambda, S^-, S^+ \geq 0$
S^-, S^+ are integers
If either $S^- \neq 0$ or $S^+ \neq 0$, then set
$\bar{X}_j = \bar{X}_j - S^-$
$\bar{Y}_j = \bar{Y}_j + S^+$
$(\bar{X}_j, \bar{Y}_j)$ is the target value satisfying the integer restrictions.

As in the case of input oriented and output oriented, the target value obtained using the algorithm $(\bar{X}_j, \bar{Y}_j)$ is a point in the integer efficient frontier. Also the algorithm reduces the input excess and output shortfall of the considered DMU to zero. So this algorithm considers the input excess and output shortfall simultaneously. This case can be very useful when input and output can simultaneously change to achieve the efficiency.

Algorithm for VRS Integer DEA Models

The various variable returns to scale (VRS) DEA models can develop by including the constraint $e\lambda = 1$ into the set of constraints. For examplethe proposed computational algorithm for the input oriented VRS integer DEA model is given by

Step 1: Measure the efficiency of the DMU_j using the classical input oriented BCC VRS model. If it is VRS efficient, then the integer target for DMU_j is $(\bar{X}_j, \bar{Y}_j)$ and stop.
Otherwise go to step 2
Step 2: Solve the MILP
$max\ S^-$
Subject to
$X\lambda + S^- = \bar{X}_j$
$Y\lambda - S^+ = \bar{Y}_j$
$e\lambda = 1$
$\lambda, S^-, S^+ \geq 0$
S^-, S^+ are integers
If $S^- \neq 0$, then set $\bar{X}_j = \bar{X}_j - S^-$ and go to step 2
Otherwise set $\bar{Y}_j = \bar{Y}_j + S^+$
Where S^- and S^+ are taken from the optimum solution of the MILP of step 2.
Then go to step 3.
Step 3: Solve the transformed MILP with new objective function $max\ S^- + S^+$.
If either $S^- \neq 0$ or $S^+ \neq 0$, then set
$\bar{X}_j = \bar{X}_j - S^-$
$\bar{Y}_j = \bar{Y}_j + S^+$
$(\bar{X}_j, \bar{Y}_j)$ is the target value satisfying the integer restrictions.
In a similar way the computational algorithm for other models can also be defined.

VI.TARGET SETTING FOR NEW DMU

The proposed algorithm can be used to set target value for a new DMU whose input (or output) is known so that with the target input output pair the DMU operates efficiently. For this assume an initial output (or input) so that with this input-output pair the DMU is inefficient. Then with the help of the proposed algorithm the target output (or input) values to make the DMU efficient can be suggested. If the input is given then the output oriented model can be used to identify the output values. Similarly if output is given, then input oriented model can be used to suggest the target input.

VII. ILLUSTRATION USING EXAMPLES

For illustration consider the situation of the evaluation of the relative efficiency of 12 hospitals in terms of two inputs, number of doctors and number of nurses, and two outputs, number of inpatients and number of outpatients a day. The data is given in table 1.

Target setting for Existing DMUs

The integer target values using the proposed CRS models for these 12 DMUs is given in table 2 and for VRS models is given in table 3.

Example for target setting for New DMU

As an example consider the above data. Suppose a new hospital M with expected outpatients 175 and inpatients 95 per day is to be set up. The algorithm can then predict the required number of nurses and doctors for the efficient operation.

By considering the initial input as (55,306),which are the maximum input values, the input oriented CRS algorithm suggest target input as (28,193). With this target input value the new DMU will place in the CRS integer efficient frontier. Also the VRS algorithm suggest target input as (31,196). With this target value the new DMU will place in the VRS integer efficient frontier. Similarly it is also possible to suggest target output for a new DMU whose input is given.

VIII. CONCLUSION

In this paper, an iterative MILP algorithm is proposed to set integer target values for inefficient DMUs. The same can also use to suggest input/output values for a new DMU so that with these values the DMU will become efficient. The algorithm is explained by considering a hospital problem data from literature with doctors and nurses as input factors and outpatients and inpatients as the output factors.

Table 1

	Hospitals	A	B	C	D	E	F	G	H	I	J	K	L
Input	Doctors	20	19	25	27	22	55	33	31	30	50	53	38
	Nurses	151	131	160	168	158	255	235	206	244	268	306	284
Output	Outpatients	100	150	160	180	94	230	220	152	190	250	260	250
	Inpatients	90	50	55	72	66	90	88	80	100	100	147	120

Table 2

Hospitals	Input oriented				Output oriented				Additive Model			
	input		output		input		output		input		output	
A	20	151	100	90	20	151	100	90	20	151	100	90
B	19	131	150	50	19	131	150	50	19	131	150	50
C	21	142	160	55	25	159	180	63	23	159	180	61
D	27	168	180	72	27	168	180	72	27	168	180	72
E	17	121	94	66	22	154	160	66	22	154	160	66
F	33	214	230	90	37	255	290	98	37	255	290	98
G	33	206	221	88	33	228	258	88	33	228	258	88
H	24	165	152	80	31	205	233	80	30	206	233	80
I	30	206	190	100	30	215	204	100	30	214	203	100
J	37	232	250	100	39	268	305	103	39	268	305	103
K	43	293	260	147	47	306	289	147	46	306	288	147
L	38	258	250	120	38	269	268	120	38	269	268	120

Table 3

Hospitals	Input oriented				Output oriented				Additive Model			
	input		output		input		output		input		output	
A	20	151	100	90	20	151	100	90	20	151	100	90
B	19	131	160	168	19	131	160	168	19	131	160	168
C	22	144	160	57	25	160	172	67	25	159	172	66
D	27	168	180	72	27	168	180	72	27	168	180	72

E	20	141	133	66	22	157	158	66	22	149	150	66
F	43	241	230	94	43	254	230	118	43	254	230	118
G	33	235	220	88	33	235	220	88	33	235	220	88
H	25	165	153	80	31	205	202	88	31	190	192	84
I	30	219	190	100	30	223	193	100	30	219	190	100
J	50	268	250	100	50	268	250	100	50	268	250	100
K	53	306	260	147	53	306	260	147	53	306	260	147
L	38	284	250	120	38	284	250	120	38	284	250	120

IX. REFERENCES

[1]Amin, G. R. (2009). Comments on Finding the Most Efficient DMUs in DEA: An Improved Integrated Model. Computers and Industrial Engineering; 56(4); 1701-1702.

[2]Amin, G. R. and Toloo, M. (2007). Finding the Most Efficient DMUs in DEA: An Improved Integrated Model. Computers and Industrial Engineering; 52(2); 71-77.

[3]Banker, R. J., Charnes, A. and Cooper, W. W. (1984). Some Models for Estimating Technical and Scale inefficiencies in Data Envelopment Analysis. Management Science 30(9), 1078-1092.

[3]Bi, G., Ding, J., Luo, Y. and Liang, L. (2011). Resource allocation and Target setting for parallel production system based on DEA. Applied Mathematical Modelling 35 4270-4280.

[3]Charnes, A., Cooper, W. W., Lewin, A. Y. and Seiford, L. M. (1994). Data Envelopment Analysis- Theory, Methodology and Applications. Kluwer Academic Publishers, Boston.

[4]Charnes, A., Cooper, W. W. and Rhodes, E. (1978). Measuring the Efficiency of Decision Making Units. European Journal of Operational Research; 2(6), 429-444.

[5]Cook, W. D. and Seiford, L. M. (2009). Data Envelopment Analysis (DEA)-Thirty Years on. European Journal of Operational Research; 192 1-17.

[6]Kazemi Matin, R. and Kuosmanen, T. (2009). Theory of Integer-valued Data Envelopment Analysis under Alternative returns to scale axioms. Omega, 37(5); 988-995.

[7]Kazemi Matin, R. (2011). A Non-radial Approach for Setting Integer-valued Targets in Data Envelopment Analysis. Journal of Optimization in Industrial Engineering; 8; 33-40.

[8]Khezrimotlagh, D., Salleh, S. and Mohsenpour, Z. (2012). A comment on theory of Integer-valued Data Envelopment Analysis. Applied Mathematical Sciences; 6(116); 5769-5774.

[9]Khezrimotlagh, D., Salleh, S. and Mohsenpour, Z. (2013). A note on Integer-valued Radial Model in DEA. Computers & Industrial Engineering; 66; 199-200 .

[10]Kuosmanen, T. and Kazemi Matin, R. (2009). Theory of integer Integer –Valued Data Envelopment Analysis. European Journal of Operational Research; 192; 658-667.

[11]Lozano, S. and Villa, G. (2006). Data Envelopment Analysis of Integer –valued inputs and outputs. Computers and Operations Research, 33; 3004-30014.

[12]Lozano, S. and Villa, G. (2007). Integer DEA Models. In Modelling Data Irregularities and Structural Complexities in Data Envelopment Analysis. pp 271-289. Springer , New York.

[13]Wu, J. and Zhou, Z. (2011). A Mixed-objective Integer DEA Model. Annals of Operations Research; 1-15.

[14]Toloo, M. and Nalchigar, S. (2009). A New Integrated DEA Model for find most BCC-efficient DMU. Applied Mathematical Modelling; 33 597-604.