C 21071 (Pages : 3)

Name..............................

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SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2017

(CUCBCSS—UG)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours

Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*

Each question carries 1 *mark.*

1. Define the limit of a complex valued function.
2. Write Cauchy's integral formula with full assumptions involved.
3. Verify whether $f(z) = 2i\bar{z}$ is analytic or not at $z = 0$?
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z-1)^2}{z^3(z^2+2)}$.
5. Is $u(x, y) = x^2 + y^2 + xy$ a harmonic function ? Justify your claim.
6. Define residue of a complex valued function.
7. Fill in the blanks : The real part of sinh $(2z)$ is ———.
8. Fill in the blanks : The locus of the points z satisfying $|z + 2i|^2 = 2|i+1|$ is a/an ———.
9. Solve for z : $5iz = 2\bar{z}$.
10. If R is the radius of convergence of $\sum a_n z^n$, find the radius of convergence of $\sum a_n z^{2n}$.
11. What do you mean by a simply connected domain ?
12. Find the value of $i^i + \log(i)$.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*

Each question carries 4 *marks.*

13. Which one is bigger : $||z_1| - |z_2||$ or $|z_1 - z_2|$. Prove your claim.
14. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.

Turn over

15. Show that $\tan^{-1}(z) = \frac{i}{2}\log\frac{i+z}{i-z}$.

16. Show that the poles of an analytic function are isolated.

17. Evaluate the line integral of $f(z) = z^2$ over the line joining $2i$ to $i-1$.

18. Find the radius of convergence of the power series :

$$\sum_{n=0}^{\infty} \frac{n!z^n}{n^n}.$$

19. Verify Cauchy-Groursat theorem for $f(z) = z^5$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Locate the poles and zeros, if any, of $f(z) = \sin(1/z)$ in the complex plane.

21. Find all the solutions of $e^z = 2$.

22. Find the residue of $f(z) = \sin(z)/z^2$ at $z = 0$ and evaluate the integral of $f(z)$ around the circle containing zero inside it.

23. Using the definition of continuity show that $\sin z$ is continuous through out the plane.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Find the real and imaginary parts of the function $f(z) = \sin(z)$.

26. Determine all the poles of the $f(z) = \sec^2 z$ lying in the disc $|z - \pi/2| \leq 3$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Evaluate $\oint_C \frac{1}{(z-a)(z-b)}$ discussing the cases of containment of the points $a \neq 0$ and $b \neq 0$ inside and outside the simple closed curve C.

28. Determine the nature of the singularities of the function $f(z) = \cos(1/z)$. Does this function have zeros ? Find them if any.

29. Find the Laurentz series expansion of $f(z) = \frac{z}{(2z-3)^2(z-2)}$ discussing the various regions of validity for the expansion.

30. Prove the converse of Cauchy-Goursat's integral theorem by fully stating the assumptions involved.

31. Find the analytic function $f(z)$ for which $u(x, y) = \text{Re}(f(z)) = e^x(x \cos y - y \sin x)$. You should express $f(z)$ finally only in terms of z.

32. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin, even though Cauchy-Riemann equations are satisfied at that point.

33. Prove the formulas for conversion Cauchy-Riemann equation into the corresponding polar form in detail.

34. Show that the derived series has the same radius of convergence as the original series.

35. Determine the locus of points of z in the complex plane satisfying the equation $|z-2| - |z-1| = 2$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) State and prove Liouvillies theorem.

(b) Prove *or* disprove : $|\cos(z)| \leq 1$ for all complex numbers z. Justify your claim.

37. (a) State and prove fundamental theorem of Algebra.

(b) Find the residues of $f(z) = \dfrac{z^2}{(z-1)^2(z-2)}$ at its poles.

38. (a) Evaluate using the method of residues : $\int_0^{2\pi} \dfrac{1}{3 + 2\cos\theta} d\theta$.

(b) Evaluate $\int_0^{\infty} \dfrac{x^2}{x^4 + a^4} dx, a > 0$.

(2 × 13 = 26 marks)