C 21070 (Pages : 4) Name..............................

Reg. No..............................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2017

(CUCBCSS—UG)

Mathematics

MAT 6B 09—REAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 mark.

1. State boundedness theorem.
2. Define absolute Minimum of a function.
3. State the preservation of interval theorem.
4. Give an example of a continuous function on $(0, \infty)$ which has neither absolute maximum nor absolute minimum.
5. Give an example of a continuous function on $A \subseteq R$, which is not uniformly continuous on A.
6. Find $\|p\|$ if $p = \{0, 0.3, 0.6, 1, 1.5, 2\}$ is a partition of the set $[0, 2]$.
7. Give an example of the 2nd kind improper integral.
8. The Radius of convergence of the power series $\sum \frac{x^n}{n!}$ is ————.
9. Define uniform convergence of a series of functions.
10. $\lim_{n\to\infty}\left(\frac{\sin(nx+n)}{n}\right) =$ ————.
11. What do you mean by uniform norm of a bounded function for $A \subseteq R$.
12. Find $\sqrt{\frac{5}{2}}$.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 marks.

13. Let $I = [a, b]$, Let $f : I \to R$ be continuous. If $K \in R$ is any number satisfying

$$\inf f(I) \leq K \leq \sup f(I),$$

then prove that there exist a number $C \in I$ such that $f(c) = k$.

Turn over

14. (a) Define Lipschitz function.

(b) If f is a Lipschitz function on $A \subseteq R$, then prove that f is uniformly continuous on A.

15. Show that the function f defined by

$$f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ 0, \text{ when } x \text{ is irrational} \end{cases}$$ rational

has no Riemann Integral over [0,1].

16. Evaluate $\int_0^\infty x^6 e^{-2x} dx$.

17. State the substitution theorem of Riemann Integration. Use it to evaluate $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}} dt$.

18. State continuous extension theorem. Use it to show that $f(x) = \sin\left(\frac{1}{x}\right)$ is not uniformly continuous on $(0, b]$, $b > 0$.

19. Determine the uniform convergence of $\sum_{n=1}^{\infty} \frac{1}{x^2 + n^2}$.

20. State and prove the Weierstrus M-Test for a series of functions.

21. If F and G are differentiable on $[a, b]$ and let $f = F'$ and $g = G'$ belongs to R $[a, b]$ then P.T. $\left[_a^b fG = FG\right]_a^b - \int_a^b Fg$.

22. State Lebesgue's Integrability Criterion for Riemann Integrability. Use it to show that every step function on $[a, b]$ is Riemann Integrable.

23. Test the convergence of $\int_1^\infty \frac{\ln x}{x^2} dx$.

24. Evaluate $\int_2^\infty \frac{(x+3)dx}{(x-1)(x^2+1)}$.

25. Prove that $\sqrt{n+1} = n\sqrt{n}$.

26. Prove that $\sqrt{n} = \int_0^\infty e^{-y^2} y^{2n-1} dy$.

(10 × 4 = 40 m

Section C

Answer any **six** *questions.*
Each question carries 7 marks.

27. State and prove Intermediate value theorem.

28. If $f \in R[a, b]$, then prove that f is bounded on $[a, b]$.

29. If $f : [a, b] \to R$ is monotone on $[a, b]$, P. T. $f \in R[a, b]$.

30. If $f : A \to R$ is uniformly continuous on $A \subseteq R$ and if (x_n) is a Cauchy sequence in A, then prove that $f(x_n)$ is a Cauchy sequence in R.

31. Discuss the convergence of the sequence $(f_n(x))$, where $f_n(x) = \frac{x^n}{x^n + 1}$, $x \in [0, 2]$.

32. State the necessary and sufficient condition for sequence (f_n) to fail to converge uniformly on A_0CR to f. Use it to test the uniform convergence of $(f_n(x))$, where $f_n(x) = \frac{x}{n}$, $x = R$.

33. If (f_n) and (g_n) are uniformly convergent sequences on $A \subseteq R$, is it imply that $(f_n)(g_n)$ is uniformly convergent on A ? Justify by an example.

34. State and prove the product theorem on Riemann Integration.

35. Prove that $\beta(m,n) = \frac{\sqrt{m}.\sqrt{n}}{\sqrt{m+n}}$, $\forall m,n > 0$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 marks.

36. (a) State and prove the Cauchy criterion for uniform convergence of a sequence of functions.

(b) Discuss the convergence of $f_n(x) = x^4$, $x \in [0, 1]$.

37. (a) State and prove uniform continuity theorem.

(b) Test the uniform continuity of $f(x) = \sqrt{x}$, $x \in [0, 2]$.

then prove that there ex...