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MODULAR AND HOMOMORPHIC PRODUCTS ON INTERVAL - VALUED INTUITIONISTIC FUZZY GRAPHS

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Abstract

In this paper, Modular product and homomorphic product on intervalvalued intuitionistic fuzzy graphs has been introduced and degree of vertices of of these new product graphs are determined. Some results involving these products are stated and proved.

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Key Words and Phrases: Interval-valued intuitionistic fuzzy graph, Strong interval - valued intuitionistic fuzzy graph, modular product, homomorphic product, degree of vertices

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1 Introduction

Zadeh [7] introduced the concept of fuzzy set in 1965 for defining uncertainty. As an extension of fuzzy sets, in 1975, Zadeh [8] introduced the notion of interval-valued fuzzy sets in which the values of the membership degrees are intervals of numbers instead of the numbers. In 1986, Atanassov introduced Intuitionistic Fuzzy Sets [6] which provides the opportunity to model the problem precisely based on the existing information and observations. After three years Atanassov and Gargov [5] proposed Interval-Valued Intuitionistic Fuzzy set (IVIFS) which is helpful to model the problem more accurately. The fuzzy graph theory was first introduced by Rosenfeld [10] in 1975. Yeh and Bang [12] also introduced fuzzy graphs independently. Fuzzy graphs are useful to represent relationships which deal with uncertainty. It has numerous applications to problems in various fields. Interval-Valued Fuzzy Graphs (IVFG) are defined by Akram and Dudek [2]in 2011. Atanassov [6] introduced the concept of intuitionistic fuzzy relations and Intuitionistic Fuzzy Graph (IFG). Shovan Dogra[11] introduced different types of products of fuzzy graphs. S.N.Mishra and A.Pal [9] introduced the product of interval valued intuitionistic fuzzy graph. Akram and Bijan Davvaz [1] introduced Strong Intuitionistic Fuzzy Graphs (SIFG). The notions of Strong Interval-Valued Intuitionistic Fuzzy Graphs (SIVIFG) are introduced by A.Mohamed Ismayil and A.Mohamed Ali [3]. This paper has been organized as follows. Preliminaries required for this study are given in section 2. In section 3 and 4, modular product and homomorphic product on interval valued intuitionistic fuzzy graphs has been defined and some of its properties are discussed

2 Preliminaries

Definition 2.1. [10] A*fuzzy graph* G is defined as a pair of functions (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ.
The underlying crisp graph of $G = (\sigma, \mu)$ is denoted by $G^* = (V, E)$ where $E \subseteq V \times V$.
Let $D[0, 1]$ be the set of all closed subintervals of the interval [0,1]. If $M \in D[0, 1]$ then it is represented by $M = [M_L, M_U]$, where M_L

and M_U are the lower and upper limits of M.

Definition 2.2. [6] An intuitionistic fuzzy graph with underlying set V is defined to be a pair $= (\sigma, \mu)$ where

- the functions $M_\sigma : V \to D[0,1]$ and $N_\sigma : V \to D[0,1]$ denote the degree of membership and non membership respectively, such that $0 \leq M_\sigma(x) + N_\sigma(x) \leq 1$ for all $x \in V$.
- the functions $M_\mu : E \subseteq V \times V \to D[0,1]$ and $N_\mu : E \subseteq V \times V \to D[0,1]$ are defined by $M_\mu((x,y)) \leq \min(M_\sigma(x), M_\sigma(y))$ and $N_\mu((x,y)) \geq \max(N_\sigma(x), N_\sigma(y))$ such that $0 \leq M_\mu((x,y)) + N_\mu((x,y)) \leq 1, \forall (x,y) \in E$.

Definition 2.3. [1] An intuitionistic fuzzy graph $G = (\sigma, \mu)$ is called strong intuitionistic fuzzy graph if
$M_\mu((x,y)) = \min(M_\sigma(x), M_\sigma(y))$ and
$N_\mu((x,y)) = \max(N_\sigma(x), N_\sigma(y)), \forall (x,y) \in E$.

Definition 2.4. [2] An interval - valued intuitionistic fuzzy graph with underlying set V is defined to be a pair $= (\sigma, \mu)$where

- the functions $M_\sigma : V \to D[0,1]$ and $N_\sigma : V \to D[0,1]$ denote the degree of membership and non membership respectively, such that $0 \leq M_\sigma(x) + N_\sigma(x) \leq 1$ for all $x \in V$.
- the functions $M_\mu : E \subseteq V \times V \to D[0,1]$ and $N_\mu : E \subseteq V \times V \to D[0,1]$ are defined by $M_{\mu L}((x,y)) \leq \min(M_{\sigma L}(x), M_{\sigma L}(y))$; $N_{\mu L}((x,y)) \geq \max(N_{\sigma L}(x), N_{\sigma L}(y))$ and
$M_{\mu U}((x,y)) \leq \min(M_{\sigma U}(x), M_{\sigma U}(y))$;
$N_{\mu U}((x,y)) \geq \max(N_{\sigma U}(x), N_{\sigma U}(y))$ such that $0 \leq M_{\mu U}((x,y)) + N_{\mu U}((x,y)) \leq 1, \forall (x,y) \in E$.

Definition 2.5. [3] An interval valued intuitionistic fuzzy graph $G = (\sigma, \mu)$ is called strong interval valued intuitionistic fuzzy graph if
$M_{\mu L}((x,y)) = \min(M_{\sigma L}(x), M_{\sigma L}(y))$;
$N_{\mu L}((x,y)) = \max(N_\sigma L(x), N_{\sigma L}(y))$
and
$M_{\mu U}((x,y)) = \min(M_{\sigma U}(x), M_{\sigma U}(y))$;
$N_{\mu U}((x,y)) = \max(N_\sigma U(x), N_{\sigma U}(y))$, $\forall (x,y) \in E$.

Definition 2.6. Let $G = (\sigma, \mu)$ be an interval valued intuitionistic fuzzy graph. For any vertex $x \in V$, degree of the vertex x is defined as an ordered pair $(d_G^-(x), d_G^+(x))$ where
$d_G^-(x) = \sum_{y \in V : xy \in E} M_{\mu L}((x,y)) - \sum_{y \in V : xy \in E} N_{\mu L}((x,y))$
and
$d_G^+(x) = \sum_{y \in V : xy \in E} M_{\mu U}((x,y)) - \sum_{y \in V : xy \in E} N_{\mu U}((x,y))$

3 Modular product on Interval-valued intuitionistic fuzzy graphs

Definition 3.1. Let σ_1 and σ_2 be interval-valued intuitionistic fuzzy subsets of V_1 and V_2 respectively.Let $\mu_1$and μ_2 be interval-valued intuitionistic fuzzy subsets of E_1 and E_2 respectively. Then Modular product $G_1 \odot G_2$ of the two interval valued intuitionistic fuzzy graphs $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ is defined as a pair $(\sigma_1 \odot \sigma_2, \mu_1 \odot \mu_2)$ where
$\sigma_1 \odot \sigma_2 = ([M_{\sigma_1 L} \odot M_{\sigma_2 L}, M_{\sigma_1 U} \odot M_{\sigma_2 U}], [N_{\sigma_1 L} \odot N_{\sigma_2 L}, N_{\sigma_1 U} \odot N_{\sigma_2 U}])$
and
$\mu_1 \odot \mu_2 = ([M_{\mu_1 L} \odot M_{\mu_2 L}, M_{\mu_1 U} \odot M_{\mu_2 U}], [N_{\mu_1 L} \odot N_{\mu_2 L}, N_{\mu_1 U} \odot N_{\mu_2 U}])$
are interval-valued intuitionistic fuzzy sets on $V = V_1 \odot V_2$ and
$E = \{\{(x_1,x_2)(y_1,y_2) : x_1y_1 \in E_1 \quad \& \quad x_2y_2 \in E_2\} \bigcup \{(x_1,x_2)(y_1,y_2) : x_1y_1 \notin E_1 \quad \& \quad x_2y_2 \notin E_2\}\}$ respectively which satisfy the following properties
$(M_{\sigma_1 L} \odot M_{\sigma_2 L})(x,y) = \min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(y)\}$
$(M_{\sigma_1 U} \odot M_{\sigma_2 U})(x,y) = \min\{M_{\sigma_1 U}(x), M_{\sigma_2 U}(y)\}$
$(N_{\sigma_1 L} \odot N_{\sigma_2 L})(x,y) = \max\{N_{\sigma_1 L}(x), N_{\sigma_2 L}(y)\}$
$(N_{\sigma_1 U} \odot N_{\sigma_2 U})(x,y) = \max\{N_{\sigma_1 U}(x), N_{\sigma_2 U}(y)\}$

$$\forall (x,y) \in V_1 \odot V_2$$

$$(M_{\mu_1 L} \odot M_{\mu_2 L})((x_1,x_2),(y_1,y_2)) = \min\{M_{\mu_1 L}(x_1y_1), M_{\mu_2 L}(x_2y_2)\}$$
$$(M_{\mu_1 U} \odot M_{\mu_2 U})((x_1,x_2),(y_1,y_2)) = \min\{M_{\mu_1 U}(x_1y_1), M_{\mu_2 U}(x_2y_2)\}$$
$$(N_{\mu_1 L} \odot N_{\mu_2 L})((x_1,x_2),(y_1,y_2)) = \max\{N_{\mu_1 L}(x_1y_1), N_{\mu_2 L}(x_2y_2)\}$$
$$(N_{\mu_1 U} \odot N_{\mu_2 U})((x_1,x_2),(y_1,y_2)) = \max\{N_{\mu_1 U}(x_1y_1), N_{\mu_2 U}(x_2y_2)\}$$

if $x_1y_1 \in E_1$ and $x_2y_2 \in E_2$

$$(M_{\mu_1 L} \odot M_{\mu_2 L})\left((x_1, x_2), (y_1, y_2)\right) = \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

$$(M_{\mu_1 U} \odot M_{\mu_2 U})\left((x_1, x_2), (y_1, y_2)\right) = \min\{M_{\sigma_1 U}(x_1), M_{\sigma_1 U}(y_1), M_{\sigma_2 U}(x_2), M_{\sigma_2 U}(y_2)\}$$

$$(N_{\mu_1 L} \odot N_{\mu_2 L})\left((x_1, x_2), (y_1, y_2)\right) = \max\{N_{\sigma_1 L}(x_1), N_{\sigma_1 L}(y_1), N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\}$$

$$(N_{\mu_1 U} \odot N_{\mu_2 U})\left((x_1, x_2), (y_1, y_2)\right) = \max\{N_{\sigma_1 U}(x_1), N_{\sigma_1 U}(y_1), N_{\sigma_2 U}(x_2), N_{\sigma_2 U}(y_2)\}$$

if $x_1 y_1 \notin E_1$ and $x_2 y_2 \notin E_2$

Proposition 3.1. *If G_1 and G_2 are strong interval-valued intuitionistic fuzzy graphs, then the Modular Product of G_1 and G_2, $G_1 \odot G_2$ is also a strong interval-valued intuitionistic fuzzy graphs.*

Proof. Let G_1 and G_2 are strong interval-valued intuitionistic fuzzy graphs.

Hence $\forall x_i y_i \in E_i, i = 1, 2$

$M_{\mu_i L}\left((x_i, y_i)\right) = \min\left(M_{\sigma_i L}(x_i), M_{\sigma_i L}(y_i)\right)$
$M_{\mu_i U}\left((x_i, y_i)\right) = \min\left(M_{\sigma_i U}(x_i), M_{\sigma_i U}(y_i)\right)$
$N_{\mu_i L}\left((x_i, y_i)\right) = \max\left(N_{\sigma_i L}(x_i), N_{\sigma_i L}(y_i)\right)$
$N_{\mu_i U}\left((x_i, y_i)\right) = \max\left(N_{\sigma_i U}(x_i), N_{\sigma_i U}(y_i)\right)$

Here $E = \{(x_1, x_2)(y_1, y_2) : x_1 y_1 \in E_1 \text{ and } x_2 y_2 \in E_2\} \cup \{(x_1, x_2)(y_1, y_2) : x_1 y_1 \notin E_1 \text{ and } x_2 y_2 \notin E_2\}$

Let$(x_1, x_2)(y_1, y_2) \in E$

Case I: $x_1 y_1 \in E_1$ and $x_2 y_2 \in E_2$

$$(M_{\mu_1 L} \odot M_{\mu_2 L})\left((x_1, x_2), (y_1, y_2)\right) = \min\{M_{\mu_1 L}(x_1 y_1), M_{\mu_2 L}(x_2 y_2)\}$$
$$= \min\left\{\min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1)\}, \min\{M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}\right\}$$

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$$= \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

and

$$\min\{(M_{\sigma_1 L} \odot M_{\sigma_2 L})(x_1, x_2), (M_{\sigma_1 L} \odot M_{\sigma_2 L})(y_1, y_2)\}$$
$$= \min\{\min\{M_{\sigma_1 L}(x_1), M_{\sigma_2 L}(x_2)\}, \min\{M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(y_2)\}\}$$
$$= \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

ie, $(M_{\mu_1 L} \odot M_{\mu_2 L})((x_1, x_2), (y_1, y_2))$
$$= \min\{(M_{\sigma_1 L} \odot M_{\sigma_2 L})(x_1, x_2), (M_{\sigma_1 L} \odot M_{\sigma_2 L})(y_1, y_2)\}$$

Similarly it can be proved that

$(M_{\mu_1 U} \odot M_{\mu_2 U})((x_1, x_2), (y_1, y_2))$
$$= \min\{(M_{\sigma_1 U} \odot M_{\sigma_2 U})(x_1, x_2), (M_{\sigma_1 U} \odot M_{\sigma_2 U})(y_1, y_2)\}$$

$(N_{\mu_1 L} \odot N_{\mu_2 L})((x_1, x_2), (y_1, y_2))$
$$= \max\{(N_{\sigma_1 L} \odot N_{\sigma_2 L})(x_1, x_2), (N_{\sigma_1 L} \odot N_{\sigma_2 L})(y_1, y_2)\}$$

$(N_{\mu_1 U} \odot N_{\mu_2 U})((x_1, x_2), (y_1, y_2))$
$$= \max\{(N_{\sigma_1 U} \odot N_{\sigma_2 U})(x_1, x_2), (N_{\sigma_1 U} \odot N_{\sigma_2 U})(y_1, y_2)\}$$

ie., in this case $G_1 \odot G_2$ is also a strong interval-valued intuitionistic fuzzy graph

Case II: $x_1 y_1 \notin E_1$ and $x_2 y_2 \notin E_2$

$(M_{\mu_1 L} \odot M_{\mu_2 L})((x_1, x_2), (y_1, y_2))$
$$= \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

and

$$\min\{(M_{\sigma_1 L} \odot M_{\sigma_2 L})(x_1, x_2), (M_{\sigma_1 L} \odot M_{\sigma_2 L})(y_1, y_2)\}$$
$$= \min\{\min\{M_{\sigma_1 L}(x_1), M_{\sigma_2 L}(x_2)\}, \min\{M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(y_2)\}\}$$
$$= \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

ie,

$(M_{\mu_1 L} \odot M_{\mu_2 L})\,((x_1, x_2), (y_1, y_2))$

$$= \min\{(M_{\sigma_1 L} \odot M_{\sigma_2 L})(x_1, x_2), (M_{\sigma_1 L} \odot M_{\sigma_2 L})(y_1, y_2)\}$$

Similarly it can be proved that

$(M_{\mu_1 U} \odot M_{\mu_2 U})\,((x_1, x_2), (y_1, y_2))$

$$= \min\{(M_{\sigma_1 U} \odot M_{\sigma_2 U})(x_1, x_2), (M_{\sigma_1 U} \odot M_{\sigma_2 U})(y_1, y_2)\}$$

$(N_{\mu_1 L} \odot N_{\mu_2 L})\,((x_1, x_2), (y_1, y_2))$

$$= \max\{(N_{\sigma_1 L} \odot N_{\sigma_2 L})(x_1, x_2), (N_{\sigma_1 L} \odot N_{\sigma_2 L})(y_1, y_2)\}$$

$(N_{\mu_1 U} \odot N_{\mu_2 U})\,((x_1, x_2), (y_1, y_2))$

$$= \max\{(N_{\sigma_1 U} \odot N_{\sigma_2 U})(x_1, x_2), (N_{\sigma_1 U} \odot N_{\sigma_2 U})(y_1, y_2)\}$$

ie., in this case also $G_1 \odot G_2$ is a strong interval-valued intuitionistic fuzzy graph

□

Proposition 3.2. *If $G_1 \odot G_2$ is a strong interval-valued intuitionistic fuzzy graph then atleat G_1 or G_2 must be strong.*

Proof. Suppose that G_1 and G_2 are not strong interval valued intuitionistic fuzzy graphs. So there exists $x_i, y_i \in E_i, i = 1, 2$ such that

$M_{\mu_i L}\,((x_i, y_i)) < \min(M_{\sigma_i L}(x_i), M_{\sigma_i L}(y_i))$
$M_{\mu_i U}\,((x_i, y_i)) < \min(M_{\sigma_i U}(x_i), M_{\sigma_i U}(y_i))$
$N_{\mu_i L}\,((x_i, y_i)) > \max(N_{\sigma_i L}(x_i), N_{\sigma_i L}(y_i))$
$N_{\mu_i U}\,((x_i, y_i)) > \max(N_{\sigma_i U}(x_i), N_{\sigma_i U}(y_i))$

$E = \{(x_1, x_2)(y_1, y_2) : x_1y_1 \in E_1 \text{ and } x_2y_2 \in E_2\} \cup \{(x_1, x_2)(y_1, y_2) : x_1y_1 \notin E_1 \text{ and } x_2y_2 \notin E_2\}$

Let $(x_1, x_2)(y_1, y_2) \in E$

If $x_1y_1 \in E_1$ and $x_2y_2 \in E_2$

$$(M_{\mu_1 L} \odot M_{\mu_2 L})\,((x_1, x_2), (y_1, y_2)) = \min\{M_{\mu_1 L}(x_1y_1), M_{\mu_2 L}(x_2y_2)\}$$
$$< \min\{\min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1)\}, \min\{M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}\}$$

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$$< \min\{M_{\sigma_1L}(x_1), M_{\sigma_1L}(y_1), M_{\sigma_2L}(x_2), M_{\sigma_2L}(y_2)\}$$

and

$$\begin{aligned}\min\{(M_{\sigma_1L} \odot M_{\sigma_2L})(x_1,x_2), (M_{\sigma_1L} \odot M_{\sigma_2L})(y_1,y_2)\} \\ = \min\{\min\{M_{\sigma_1L}(x_1), M_{\sigma_2L}(x_2)\}, \min\{M_{\sigma_1L}(y_1), M_{\sigma_2L}(y_2)\}\} \\ = \min\{M_{\sigma_1L}(x_1), M_{\sigma_1L}(y_1), M_{\sigma_2L}(x_2), M_{\sigma_2L}(y_2)\}\end{aligned}$$

ie,

$$\begin{aligned}(M_{\mu_1L} \odot M_{\mu_2L})((x_1,x_2),(y_1,y_2)) \\ < \min\{(M_{\sigma_1L} \odot M_{\sigma_2L})(x_1,x_2), (M_{\sigma_1L} \odot M_{\sigma_2L})(y_1,y_2)\}\end{aligned}$$

Similarly, it can be proved that

$$\begin{aligned}(M_{\mu_1U} \odot M_{\mu_2U})((x_1,x_2),(y_1,y_2)) \\ < \min\{(M_{\sigma_1U} \odot M_{\sigma_2U})(x_1,x_2), (M_{\sigma_1U} \odot M_{\sigma_2U})(y_1,y_2)\} \\ (N_{\mu_1L} \odot N_{\mu_2L})((x_1,x_2),(y_1,y_2)) \\ > \max\{(N_{\sigma_1L} \odot N_{\sigma_2L})(x_1,x_2), (N_{\sigma_1L} \odot N_{\sigma_2L})(y_1,y_2)\} \\ (N_{\mu_1U} \odot N_{\mu_2U})((x_1,x_2),(y_1,y_2)) \\ > \max\{(N_{\sigma_1U} \odot N_{\sigma_2U})(x_1,x_2), (N_{\sigma_1U} \odot N_{\sigma_2U})(y_1,y_2)\}\end{aligned}$$

ie., in this case $G_1 \odot G_2$ is not a strong interval-valued intuitionistic fuzzy graph

□

Definition 3.2. For any vertex $(x_1,x_2) \in V_1 \odot V_2$ in $G_1 \odot G_2$, degree of the vertex (x_1,x_2) is defined as an ordered pair $\left(d^-_{G_1 \odot G_2}(x_1,x_2), d^+_{G_1 \odot G_2}(x_1,x_2)\right)$ where

$$d^{-}_{G_1 \odot G_2}(x_1, x_2) = \left(\sum_{\substack{(y_1,y_2)\in V_1\odot V_2: \\ x_1y_1\in E_1, x_2y_2\in E_2}} \min\{M_{\mu_1 L}(x_1, y_1), M_{\mu_2 L}(x_2, y_2)\} \right.$$

$$\left. + \sum_{\substack{(y_1,y_2)\in V_1\odot V_2: \\ x_1y_1\notin E_1, x_2y_2\notin E_2}} \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\} \right)$$

$$- \left(\sum_{\substack{(y_1,y_2)\in V_1\odot V_2: \\ x_1y_1\in E_1, x_2y_2\in E_2}} \max\{N_{\mu_1 L}(x_1, y_1), N_{\mu_2 L}(x_2, y_2)\} \right.$$

$$\left. + \sum_{\substack{(y_1,y_2)\in V_1\odot V_2: \\ x_1y_1\notin E_1, x_2y_2\notin E_2}} \max\{N_{\sigma_1 L}(x_1), N_{\sigma_1 L}(y_1), N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\} \right)$$

and

$$d^{+}_{G_1 \odot G_2}(x_1, x_2) = \left(\sum_{\substack{(y_1,y_2)\in V_1\odot V_2: \\ x_1y_1\in E_1, x_2y_2\in E_2}} \min\{M_{\mu_1 U}(x_1, y_1), M_{\mu_2 U}(x_2, y_2)\} \right.$$

$$\left. + \sum_{\substack{(y_1,y_2)\in V_1\odot V_2: \\ x_1y_1\notin E_1, x_2y_2\notin E_2}} \min\{M_{\sigma_1 U}(x_1), M_{\sigma_1 U}(y_1), M_{\sigma_2 U}(x_2), M_{\sigma_2 U}(y_2)\} \right)$$

$$- \left(\sum_{\substack{(y_1,y_2)\in V_1\odot V_2: \\ x_1y_1\in E_1, x_2y_2\in E_2}} \max\{N_{\mu_1 U}(x_1, y_1), N_{\mu_2 U}(x_2, y_2)\} \right.$$

$$\left. + \sum_{\substack{(y_1,y_2)\in V_1\odot V_2: \\ x_1y_1\notin E_1, x_2y_2\notin E_2}} \max\{N_{\sigma_1 U}(x_1), N_{\sigma_1 U}(y_1), N_{\sigma_2 U}(x_2), N_{\sigma_2 U}(y_2)\} \right)$$

Definition 3.3. Let $\sigma_1 = ([M_{\sigma_1 L}, M_{\sigma_1 U}], [N_{\sigma_1 L}, N_{\sigma_1 U}])$ and $\sigma_2 = ([M_{\sigma_2 L}, M_{\sigma_2 U}], [N_{\sigma_2 L}, N_{\sigma_2 U}])$ be interval valued intuitionistic fuzzy subsets of $V_1(G_1)$ and $V_2(G_2)$ respectively. Then $\sigma_1 \leq \sigma_2$ if and only if $M_{\sigma_1 L}(x) \leq M_{\sigma_2 L}(y), M_{\sigma_1 U}(x) \leq M_{\sigma_2 U}(y)$ and $N_{\sigma_1 L}(x) \geq N_{\sigma_2 L}(y), N_{\sigma_1 U}(x) \geq N_{\sigma_2 U}(y)$, $\forall x \in V_1$ and $\forall y \in V_2$

Proposition 3.3. *Let $\sigma_1$and σ_2 be interval valued intuitionistic fuzzy subsets of V_1 and V_2 respectively where $\sigma_1 \leq \sigma_2$. Let $\mu_1$and μ_2 be interval valued intuitionistic fuzzy subsets of E_1 and E_2 respectively . Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ and $G = G_1 \odot G_2$. Then the following equalities hold.*

$$d^-_{G_1 \odot G_2}(x_1, x_2) =$$

$$\left(\sum_{\substack{(y_1,y_2) \in V_1 \odot V_2: \\ x_1 y_1 \in E_1, x_2 y_2 \in E_2}} M_{\mu_1 L}(x_1, y_1) + \sum_{\substack{(y_1,y_2) \in V_1 \odot V_2: \\ x_1 y_1 \notin E_1, x_2 y_2 \notin E_2}} \min \{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1)\} \right)$$

$$- \left(\sum_{\substack{(y_1,y_2) \in V_1 \odot V_2: \\ x_1 y_1 \in E_1, x_2 y_2 \in E_2}} N_{\mu_2 L}(x_2, y_2) + \sum_{\substack{(y_1,y_2) \in V_1 \odot V_2: \\ x_1 y_1 \notin E_1, x_2 y_2 \notin E_2}} \max \{N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\} \right)$$

and

$$d^+_{G_1 \odot G_2}(x_1, x_2) =$$

$$\left(\sum_{\substack{(y_1,y_2) \in V_1 \odot V_2: \\ x_1 y_1 \in E_1, x_2 y_2 \in E_2}} M_{\mu_1 U}(x_1, y_1) + \sum_{\substack{(y_1,y_2) \in V_1 \odot V_2: \\ x_1 y_1 \notin E_1, x_2 y_2 \notin E_2}} \min \{M_{\sigma_1 U}(x_1), M_{\sigma_1 U}(y_1)\} \right)$$

$$- \left(\sum_{\substack{(y_1,y_2) \in V_1 \odot V_2: \\ x_1 y_1 \in E_1, x_2 y_2 \in E_2}} N_{\mu_2 U}(x_2, y_2) + \sum_{\substack{(y_1,y_2) \in V_1 \odot V_2: \\ x_1 y_1 \notin E_1, x_2 y_2 \notin E_2}} \max \{N_{\sigma_2 U}(x_2), N_{\sigma_2 U}(y_2)\} \right)$$

Proposition 3.4. *Let $\sigma_1$and σ_2 be interval valued intuitionistic fuzzy subsets of V_1 and V_2 respectively where $\sigma_1 \leq \sigma_2$. Let $\mu_1$and μ_2 be interval valued intuitionistic fuzzy subsets of E_1 and E_2 respectively . Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be complete interval valued intuitionistic fuzzy graphs and let $G = G_1 \odot G_2$. Then the following equalities hold.*
$d^-_{G_1 \odot G_2}(x_1, x_2) = \sum_{y_1 \in V_1} M_{\mu_1 L}(x_1, y_1) - \sum_{y_2 \in V_2} N_{\mu_2 L}(x_2, y_2)$
and
$d^+_{G_1 \odot G_2}(x_1, x_2) = \sum_{y_1 \in V_1} M_{\mu_1 U}(x_1, y_1) - \sum_{y_2 \in V_2} N_{\mu_2 U}(x_2, y_2)$

4 Homomorphic product on Interval-valued intuitionistic fuzzy graphs

Definition 4.1. Let σ_1 and σ_2 be interval-valued intuitionistic fuzzy subsets of V_1 and V_2 respectively. Let $\mu_1$and μ_2 be interval-valued intuitionistic fuzzy subsets of E_1 and E_2 respectively. Then homomorphic product $G_1 \diamond G_2$ of the two interval -valued intuitionistic fuzzy graphs $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ is defined as a pair $(\sigma_1 \diamond \sigma_2, \mu_1 \diamond \mu_2)$ where
$\sigma_1 \diamond \sigma_2 = ([M_{\sigma_1 L} \diamond M_{\sigma_2 L}, M_{\sigma_1 U} \diamond M_{\sigma_2 U}], [N_{\sigma_1 L} \diamond N_{\sigma_2 L}, N_{\sigma_1 U} \diamond N_{\sigma_2 U}])$and
$\mu_1 \diamond \mu_2 = ([M_{\mu_1 L} \diamond M_{\mu_2 L}, M_{\mu_1 U} \diamond M_{\mu_2 U}], [N_{\mu_1 L} \diamond N_{\mu_2 L}, N_{\mu_1 U} \diamond N_{\mu_2 U}])$
are interval-valued intuitionistic fuzzy sets on $V = V_1 \diamond V_2$ and $E = \{\{(x, x_2)(x, y_2) : x \in V_1 \ \& \ x_2 y_2 \in E_2\} \bigcup \{(x_1, x_2)(y_1, y_2) : x_1 y_1 \in E_1 \ \& \ x_2 y_2 \notin E_2\}\}$respectively which satisfy the following properties
$(M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x, y) = \min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(y)\}$
$(M_{\sigma_1 U} \diamond M_{\sigma_2 U})(x, y) = \min\{M_{\sigma_1 U}(x), M_{\sigma_2 U}(y)\}$
$(N_{\sigma_1 L} \diamond N_{\sigma_2 L})(x, y) = \max\{N_{\sigma_1 L}(x), N_{\sigma_2 L}(y)\}$
$(N_{\sigma_1 U} \diamond N_{\sigma_2 U})(x, y) = \max\{N_{\sigma_1 U}(x), N_{\sigma_2 U}(y)\}$

$$\forall (x, y) \in V_1 \diamond V_2$$

$(M_{\mu_1 L} \diamond M_{\mu_2 L})\left((x, x_2), (x, y_2)\right) = \min\{M_{\sigma_1 L}(x), M_{\mu_2 L}(x_2 y_2)\}$
$(M_{\mu_1 U} \diamond M_{\mu_2 U})\left((x, x_2), (x, y_2)\right) = \min\{M_{\sigma_1 U}(x), M_{\mu_2 U}(x_2 y_2)\}$
$(N_{\mu_1 L} \diamond N_{\mu_2 L})\left(x, x_2), (x, y_2)\right) = \max\{N_{\sigma_1 L}(x), N_{\mu_2 L}(x_2 y_2)\}$
$(N_{\mu_1 U} \diamond N_{\mu_2 U})\left((x, x_2), (x, y_2)\right) = \max\{N_{\sigma_1 U}(x), N_{\mu_2 U}(x_2 y_2)\}$

if $x \in V_1$ and $x_2y_2 \in E_2$

$$(M_{\mu_1L} \diamond M_{\mu_2L})((x_1, x_2), (y_1, y_2)) = \min\{M_{\mu_1L}(x_1y_1), M_{\sigma_2L}(x_2), M_{\sigma_2L}(y_2)\}$$

$$(M_{\mu_1U} \diamond M_{\mu_2U})((x_1, x_2), (y_1, y_2)) = \min\{M_{\mu_1U}(x_1y_1), M_{\sigma_2U}(x_2), M_{\sigma_2U}(y_2)\}$$

$$(N_{\mu_1L} \diamond N_{\mu_2L})((x_1, x_2), (y_1, y_2)) = \max\{N_{\mu_1L}(x_1y_1), N_{\sigma_2L}(x_2), N_{\sigma_2L}(y_2)\}$$

$$(N_{\mu_1U} \diamond N_{\mu_2U})((x_1, x_2), (y_1, y_2)) = \max\{N_{\mu_1U}(x_1y_1), N_{\sigma_2U}(x_2), N_{\sigma_2U}(y_2)\}$$

if $x_1y_1 \in E_1$ and $x_2y_2 \notin E_2$

Proposition 4.1. *If G_1 and G_2 are strong interval-valued intuitionistic fuzzy graphs, then the Homomorphic Product of G_1 and G_2, $G_1 \diamond G_2$ is also a strong interval-valued intuitionistic fuzzy graphs.*

Proof. Let G_1 and G_2 are strong interval-valued intuitionistic fuzzy graphs.

Hence $\forall x_iy_i \in E_i, i = 1, 2$

$M_{\mu_iL}((x_i, y_i)) = \min(M_{\sigma_iL}(x_i), M_{\sigma_iL}(y_i))$
$M_{\mu_iU}((x_i, y_i)) = \min(M_{\sigma_iU}(x_i), M_{\sigma_iU}(y_i))$
$N_{\mu_iL}((x_i, y_i)) = \min(N_{\sigma_iL}(x_i), N_{\sigma_iL}(y_i))$
$N_{\mu_iU}((x_i, y_i)) = \min(N_{\sigma_iU}(x_i), N_{\sigma_iU}(y_i))$

Here $E = \{(x, x_2)(x, y_2) : x \in V_1 \text{ and } x_2y_2 \in E_2\} \cup \{(x_1, x_2)(y_1, y_2) : x_1y_1 \in E_1 \text{ and } x_2y_2 \notin E_2\}$

Let$(x_1, x_2)(y_1, y_2) \in E$

Case I: $x_1 = y_1 = x$ and $x_2y_2 \in E_2$

$$(M_{\mu_1L} \diamond M_{\mu_2L})((x, x_2)(x, y_2)) = \min\{M_{\sigma_1L}(x), M_{\mu_2L}(x_2y_2)\}$$

$$= \min\{M_{\sigma_1 L}(x), \min\{M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}\}$$
$$= \min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

and

$$\min\{(M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x, x_2), (M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x, y_2)\}$$
$$= \min\{\min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(x_2)\}, \min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(y_2)\}\}$$
$$= \min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

ie,

$$(M_{\mu_1 L} \diamond M_{\mu_2 L})((x, x_2), (x, y_2))$$
$$= \min\{(M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x, x_2), (M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x, y_2)\}$$

Similarly it can be proved that

$$(M_{\mu_1 U} \diamond M_{\mu_2 U})((x, x_2), (x, y_2))$$
$$= \min\{(M_{\sigma_1 U} \diamond M_{\sigma_2 U})(x, x_2), (M_{\sigma_1 U} \diamond M_{\sigma_2 U})(x, y_2)\}$$
$$(N_{\mu_1 L} \diamond N_{\mu_2 L})((x, x_2), (x, y_2))$$
$$= \max\{(N_{\sigma_1 L} \diamond N_{\sigma_2 L})(x, x_2), (N_{\sigma_1 L} \diamond N_{\sigma_2 L})(x, y_2)\}$$
$$(N_{\mu_1 U} \diamond N_{\mu_2 U})((x, x_2), (x, y_2))$$
$$= \max\{(N_{\sigma_1 U} \diamond N_{\sigma_2 U})(x, x_2), (N_{\sigma_1 U} \diamond N_{\sigma_2 U})(x, y_2)\}$$

ie., in this case $G_1 \diamond G_2$ is also a strong interval-valued intuitionistic fuzzy graph

Case II: $x_1 y_1 \in E_1$ and $x_2 y_2 \notin E_2$

$$(M_{\mu_1 L} \diamond M_{\mu_2 L})((x_1, x_2), (y_1, y_2))$$
$$= \min\{M_{\mu_1 L}(x_1 y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$
$$= \min\{\min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1)\}, M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$
$$= \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

and

$$\min\{(M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x_1, x_2), (M_{\sigma_1 L} \diamond M_{\sigma_2 L})(y_1, y_2)\}$$
$$= \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

ie, $(M_{\mu_1 L} \diamond M_{\mu_2 L})((x_1, x_2), (y_1, y_2))$

$$= \min\{(M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x_1, x_2), (M_{\sigma_1 L} \diamond M_{\sigma_2 L})(y_1, y_2)\}$$

Similarly it can be proved that

$$(M_{\mu_1 U} \diamond M_{\mu_2 U})((x_1, x_2), (y_1, y_2))$$
$$= \min\{(M_{\sigma_1 U} \diamond M_{\sigma_2 U})(x_1, x_2), (M_{\sigma_1 U} \diamond M_{\sigma_2 U})(y_1, y_2)\}$$
$$(N_{\mu_1 L} \diamond N_{\mu_2 L})((x_1, x_2), (y_1, y_2))$$
$$= \max\{(N_{\sigma_1 L} \diamond N_{\sigma_2 L})(x_1, x_2), (N_{\sigma_1 L} \diamond N_{\sigma_2 L})(y_1, y_2)\}$$
$$(N_{\mu_1 U} \diamond N_{\mu_2 U})((x_1, x_2), (y_1, y_2))$$
$$= \max\{(N_{\sigma_1 U} \diamond N_{\sigma_2 U})(x_1, x_2), (N_{\sigma_1 U} \diamond N_{\sigma_2 U})(y_1, y_2)\}$$

ie., in this case also $G_1 \diamond G_2$ is a strong interval-valued intuitionistic fuzzy graph

□

Proposition 4.2. *If $G_1 \diamond G_2$ is a strong interval-valued intuitionistic fuzzy graph then atleast G_1 or G_2 must be strong.*

Proof. Suppose that G_1 and G_2 are not strong interval valued intuitionistic fuzzy graphs. So there exists $x_i, y_i \in E_i, i = 1, 2$ such that

$M_{\mu_i L}((x_i, y_i)) < \min(M_{\sigma_i L}(x_i), M_{\sigma_i L}(y_i))$
$M_{\mu_i U}((x_i, y_i)) < \min(M_{\sigma_i U}(x_i), M_{\sigma_i U}(y_i))$
$N_{\mu_i L}((x_i, y_i)) > \max(N_{\sigma_i L}(x_i), N_{\sigma_i L}(y_i))$
$N_{\mu_i U}((x_i, y_i)) > \max(N_{\sigma_i U}(x_i), N_{\sigma_i U}(y_i))$

Here $E = \{(x, x_2)(x, y_2) : x \in V_1 \text{ and } x_2 y_2 \in E_2\} \cup \{(x_1, x_2)(y_1, y_2) : x_1 y_1 \in E_1 \text{ and } x_2 y_2 \notin E_2\}$

Let$(x_1, x_2)(y_1, y_2) \in E$

Case I: $x_1 = y_1 = x$ and $x_2 y_2 \in E_2$

$$(M_{\mu_1 L} \diamond M_{\mu_2 L})((x, x_2)(x, y_2)) = \min\{M_{\sigma_1 L}(x), M_{\mu_2 L}(x_2 y_2)\}$$
$$< \min\{M_{\sigma_1 L}(x), \min\{M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}\}$$
$$< \min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

but

$$\min\{(M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x, x_2), (M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x, y_2)\}$$

$$= \min\{\min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(x_2)\}, \min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(y_2)\}\}$$
$$= \min\{M_{\sigma_1 L}(x), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

ie,

$$(M_{\mu_1 L} \diamond M_{\mu_2 L})((x, x_2), (x, y_2))$$
$$< \min\{(M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x, x_2), (M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x, y_2)\}$$

Similarly it can be proved that

$$(M_{\mu_1 U} \diamond M_{\mu_2 U})((x, x_2), (x, y_2))$$
$$< \min\{(M_{\sigma_1 U} \diamond M_{\sigma_2 U})(x, x_2), (M_{\sigma_1 U} \diamond M_{\sigma_2 U})(x, y_2)\}$$
$$(N_{\mu_1 L} \diamond N_{\mu_2 L})((x, x_2), (x, y_2))$$
$$> \max\{(N_{\sigma_1 L} \diamond N_{\sigma_2 L})(x, x_2), (N_{\sigma_1 L} \diamond N_{\sigma_2 L})(x, y_2)\}$$
$$(N_{\mu_1 U} \diamond N_{\mu_2 U})((x, x_2), (x, y_2))$$
$$> \max\{(N_{\sigma_1 U} \diamond N_{\sigma_2 U})(x, x_2), (N_{\sigma_1 U} \diamond N_{\sigma_2 U})(x, y_2)\}$$

ie., in this case $G_1 \diamond G_2$ is not a strong interval-valued intuitionistic fuzzy graph

Case II: $x_1y_1 \in E_1$ and $x_2y_2 \notin E_2$

$$(M_{\mu_1 L} \diamond M_{\mu_2 L})((x_1, x_2), (y_1, y_2))$$

$$= \min\{M_{\mu_1 L}(x_1y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$
$$< \min\{\min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1)\}, M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$
$$< \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

and

$$\min\{(M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x_1, x_2), (M_{\sigma_1 L} \diamond M_{\sigma_2 L})(y_1, y_2)\}$$

$$= \min\{M_{\sigma_1 L}(x_1), M_{\sigma_1 L}(y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\}$$

ie, $(M_{\mu_1 L} \diamond M_{\mu_2 L})((x_1, x_2), (y_1, y_2))$

$$< \min\{(M_{\sigma_1 L} \diamond M_{\sigma_2 L})(x_1, x_2), (M_{\sigma_1 L} \diamond M_{\sigma_2 L})(y_1, y_2)\}$$

Similarly it can be proved that

$$(M_{\mu_1 U} \diamond M_{\mu_2 U})\left((x_1,x_2),(y_1,y_2)\right) < \min\{(M_{\sigma_1 U} \diamond M_{\sigma_2 U})(x_1,x_2),(M_{\sigma_1 U} \diamond M_{\sigma_2 U})(y_1,y_2)\}$$

$$(N_{\mu_1 L} \diamond N_{\mu_2 L})\left((x_1,x_2),(y_1,y_2)\right) > \max\{(N_{\sigma_1 L} \diamond N_{\sigma_2 L})(x_1,x_2),(N_{\sigma_1 L} \diamond N_{\sigma_2 L})(y_1,y_2)\}$$

$$(N_{\mu_1 U} \diamond N_{\mu_2 U})\left((x_1,x_2),(y_1,y_2)\right) > \max\{(N_{\sigma_1 U} \diamond N_{\sigma_2 U})(x_1,x_2),(N_{\sigma_1 U} \diamond N_{\sigma_2 U})(y_1,y_2)\}$$

ie., in this case also $G_1 \diamond G_2$ is not a strong interval-valued intuitionistic fuzzy graph

□

Definition 4.2. For any vertex $(x_1,x_2) \in V_1 \diamond V_2$ in $G_1 \diamond G_2$, degree of the vertex (x_1,x_2) is defined as an ordered pair $\left(d^-_{G_1 \diamond G_2}(x_1,x_2), d^+_{G_1 \diamond G_2}(x_1,x_2)\right)$ where

$$d^-_{G_1 \diamond G_2}(x_1,x_2) = \left(\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2: \\ x_1=y_1\in V_1; x_2y_2\in E_2}} \min\{M_{\sigma_1 L}(x_1), M_{\mu_2 L}(x_2,y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1\diamond V_2: \\ x_1y_1\in E_1; x_2y_2\notin E_2}} \min\{M_{\mu_1 L}(x_1y_1), M_{\sigma_2 L}(x_2), M_{\sigma_2 L}(y_2)\} \right) - \left(\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2: \\ x_1=y_1\in V_1; x_2y_2\in E_2}} \max\{N_{\sigma_1 L}(x_1), N_{\mu_2 L}(x_2,y_2)\} + \sum_{\substack{(y_1,y_2)\in V_1\diamond V_2: \\ x_1y_1\in E_1; x_2y_2\notin E_2}} \max\{N_{\mu_1 L}(x_1y_1), N_{\sigma_2 L}(x_2), N_{\sigma_2 L}(y_2)\} \right)$$

and

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$$d^{+}_{G_1\diamond G_2}(x_1,x_2)=\left(\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1=y_1\in V_1;x_2y_2\in E_2}}\min\{M_{\sigma_1 U}(x_1),M_{\mu_2 U}(x_2,y_2)\}\right.$$

$$\left.+\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1y_1\in E_1;x_2y_2\notin E_2}}\min\{M_{\mu_1 U}(x_1y_1),M_{\sigma_2 U}(x_2),M_{\sigma_2 U}(y_2)\}\right)$$

$$-\left(\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1=y_1\in V_1;x_2y_2\in E_2}}\max\{N_{\sigma_1 U}(x_1),N_{\mu_2 U}(x_2,y_2)\}\right.$$

$$\left.+\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1y_1\in E_1;x_2y_2\notin E_2}}\max\{N_{\mu_1 U}(x_1y_1),N_{\sigma_2 U}(x_2),N_{\sigma_2 U}(y_2)\}\right)$$

Proposition 4.3. *Let σ_1 and σ_2 be interval valued intuitionistic fuzzy subsets of V_1 and V_2 respectively where $\sigma_1\le\sigma_2$. Let μ_1 and μ_2 be interval valued intuitionistic fuzzy subsets of E_1 and E_2 respectively . Let $G_1=(\sigma_1,\mu_1)$ and $G_2=(\sigma_2,\mu_2)$ and $G=G_1\diamond G_2$. Then the following equalities hold.*

$$d^{-}_{G_1\diamond G_2}(x_1,x_2)$$

$$=\left(\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1=y_1\in V_1;x_2y_2\in E_2}}M_{\sigma_1 L}(x_1)+\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1y_1\in E_1;x_2y_2\notin E_2}}M_{\mu_1 L}(x_1y_1)\right)$$

$$-\left(\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1=y_1\in V_1;x_2y_2\in E_2}}N_{\sigma_1 L}(x_1)+\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1y_1\in E_1;x_2y_2\notin E_2}}N_{\mu_1 L}(x_1y_1)\right)$$

and

$$d^{+}_{G_1\diamond G_2}(x_1,x_2)$$

$$=\left(\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1=y_1\in V_1;x_2y_2\in E_2}}M_{\sigma_1 U}(x_1)+\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1y_1\in E_1;x_2y_2\notin E_2}}M_{\mu_1 U}(x_1y_1)\right)$$

$$-\left(\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1=y_1\in V_1;x_2y_2\in E_2}}N_{\sigma_1 U}(x_1)+\sum_{\substack{(y_1,y_2)\in V_1\diamond V_2:\\ x_1y_1\in E_1;x_2y_2\notin E_2}}N_{\mu_1 U}(x_1y_1)\right)$$

5 Conclusion

In this paper, modular product and homomorphic product on interval valued intuitionistic fuzzy graphs and some properties of these products on strong interval valued intuitionistic fuzzy graphs are discussed. We have determined the degree of vertices of these product graphs under certain conditions. Our future plan is to extend our research to some other operations on interval valued intuitionistic fuzzy graphs.

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