**15U605** (Pages: 3) Name: ……………………

Reg. No……………………

**SIXTH SEMESTER B.Sc. DEGREE EXAMINATION- MARCH 2018**

(CUCBCSS – UG)

**CC15U MAT6 E02 - LINEAR PROGRAMMING**

Mathematics – Elective Course

(2015 Admission)

Time: Three Hours Maximum: 80 Marks

## Section A

Answer *all* questions. Each question carries 1 mark.

1. Define Convex hull of a set.
2. Show that a hyperplane in $\mathbb{R}^n$ is convex.
3. Write down the standard form of LPP
4. Shade the region of $2x - 3y \geq 6, y \leq 1$
5. Define optimum basic feasible solution of a Linear Programming Problem.
6. Write down the condition for optimality in simplex method.
7. When does simplex method indicates existence of more than one solution for LPP?
8. Define penality in Charne's method.
9. If the primal has three equations as constrains predict the nature of the dual variables.
10. What is meant by an unbalanced transportation problem?
11. What are the possible solutions to the variables in an assignment problem?
12. Write down the LPP for a general assignment problem.

**(12 x 1 = 12 Marks)**

## Section B

Answer any *nine* questions. Each question carries 2 marks.

13. Find the optimum solution by graphical method :

$$\text{Maximize } z = 2x + 3y$$
$$\text{subject to } \quad 0 \leq x \leq 3$$
$$0 \leq y \leq 2$$

14. Show that the line segment is convex.
15. State and prove Minimax theorem.
16. Distinguish between slack and surplus variables with illustration.
17. Reduce to canonical form of the LPP :

$$\text{Maximize } Z = 3x_1 + 2x_2$$
$$\text{subject to } 3x_1 + 2x_2 \leq 6,$$
$$x_1 - x_2 = -1, \; x_1, x_2 \geq 0$$



**Turn Over**

18. State and prove fundamental theorem of linear programming.
19. Write down the relationship between primal and dual problem.
20. Find dual of the given primal problem

$$\text{Maximize } z = x_1 - x_2 + 3x_3$$
$$\text{Subject to } x_1 + x_2 - x_3 \leq 10,$$
$$2x_1 - x_3 \leq 2, x_1,$$
$$x_1, x_2 \geq 0,$$
$$x_3 \text{unrestricted}$$

21. Explain Matrix Minima Method.
22. Show that in a transportation problem a basic feasible solution will consist of at most $m + n - 1$ positive variables.
23. Explain Hungarian Method.
24. Four machines are to be assigned for four jobs. The cost matrix is given. Find the proper assignment

Machine

| | | | | |
|---|---|---|---|---|
| | 20 | 25 | 22 | 28 |
| Job | 15 | 18 | 23 | 17 |
| | 19 | 17 | 21 | 24 |
| | 25 | 23 | 24 | 24 |

**(9 x 2 = 18 Marks)**

**Section C**

Answer any *six* questions. Each question carries 5 marks.

25. A farmer has 1000 acres of land on which he can grow corn, wheat or soyabeans. Each acre of corn cost Rs. 100 for preparation, requires 7 man-days of work and yield a profit of Rs. 30. An acre of wheat cost Rs. 120 to prepare, requires 10 man-days of work and yields a profit of Rs. 40. An acre of soyabean cost Rs. 70 to prepare , requires 8 man-days of work and yield a profit of Rs. 20. The farmer has Rs. 100000 for preparation And 8000 man-days of work. Set up the linear programming equation for the problem.
26. Find the minimum value of $Z = 6x_1 + x_2$ subject to the constraints

$$2x_1 + x_2 \geq 3$$
$$x_1 - x_2 \geq 0$$
$$x_1, x_2 \geq 0$$

27. Show that $B = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \ / \ x_1^3 + x_2^3 + x_3^3 \leq 1\}$ is convex.
28. Check whether basic solutions are degenerate or not by sub matrix method:

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

29. Explain two phase method for solving linear programming problem.
30. Show that dual of the dual is primal.
31. Prove that in a transportation problem every loop has even number of cells.
32. Find initial basic feasible solution by VAM method

| | To | | | |
|---|---|---|---|---|
| From | 16 | 19 | 12 | 14 |
| | 22 | 13 | 19 | 16 Availability |
| | 14 | 28 | 8 | 12 |
| | 10 | 15 | 17 | |
| | | Requirement | | |

33. Four persons are to be assigned four job. Cost matrix is given below. Find an optimal assignment.

| | Job | | | |
|---|---|---|---|---|
| Person | 20 | 25 | 22 | 28 |
| | 15 | 18 | 23 | 17 |
| | 19 | 17 | 21 | 24 |
| | 25 | 23 | 24 | 24 |

**(6 x 5 = 30 Marks)**

## Section D

Answer any ***two*** questions. Each question carries 10 marks.

34. Let $A \subset \mathbb{R}^n$ be any set. Then prove that the convex hull of A, $< A >$ is the set of all finite convex combination of vectors in $A$.
35. Find dual of the following LPP. Hence find the solution using dual

$$\text{Maximize } z = 2x_1 + x_2$$
$$\text{subject to } 3x_1 + 5x_2 \leq 15,$$
$$6x_1 + 2x_2 \leq 24,$$
$$x_1, x_2 \geq 0$$

36. Solve

| | To | | | |
|---|---|---|---|---|
| From | 4 | 3 | 2 | 10 |
| | 1 | 5 | 0 | 13 Availability |
| | 3 | 8 | 6 | 12 |
| | 8 | 5 | 4 | |
| | | Requirement | | |

**(2 x 10 = 30 Marks)**

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