16U601 (Pages: 2) Name……..……..……….

Reg. No……………..………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2019

(Regular/Supplementary/Improvement)

(CUCBCSS-UG)

CC15U MAT6 B09 - REAL ANALYSIS

Mathematics - Core Course

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. State whether the following statement is True or False : $f(x) = e^x$ has neither an absolute maximum nor an absolute minimum on the set $(0, 2)$. Justify.
2. Does the function $g(x) = x^3 - 3x$ has a root in $[1, 2]$?
3. Give an example for a continuous function which is not uniformly continuous on $(1, 2)$
4. Give a partition for the interval $I = [1, 8]$
5. Give an example for a function $f : [0, \pi] \to \mathbb{R}$ such that $f \notin \mathcal{R}[0, \pi]$
6. Consider the Signum function Sgn on $[-5, 5]$ and so find $\int_{-5}^{5} \text{Sgn x dx}$
7. Define uniform convergence of a sequence of functions $\{f_n(x)\}$
8. Find the value of $\lim_{n\to\infty}\{f_n(x)\}$ where $f_n(x) = x^n$ on $(-1, 1)$
9. State Weierstrass M- Test.
10. Define an improper integral of second kind with a suitable example.
11. Find the value of $\beta(3, 4)$
12. What is the value of $\Gamma\left(\frac{1}{2}\right)$

(12 x 1 = 12 Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. State Bolzano's Intermediate Value Theorem.
14. Define absolute maximum point and absolute minimum point for $f: A \longrightarrow \mathbb{R};\ A \subseteq \mathbb{R}$
15. Define Uniform Continuity of a function $f: A \longrightarrow \mathbb{R};\ A \subseteq \mathbb{R}$ with suitable examples.
16. Define the Riemann sum and Riemann Integral of a function f on an interval $[a, b]$ corresponding to a tagged partition $\dot{P}$
17. If $f \in \mathcal{R}[a, b]$ then prove that the value of the integral is uniquely determined.
18. State the substitution theorem of Riemann integration. Use it to evaluate $\int_1^4 \frac{\cos\sqrt{t}}{\sqrt{t}}$
19. Use the Fundamental Theorem of calculus to evaluate $\int_2^5 (2x^2 + 3x + 1)\ dx$
20. Show that the sequence $\{f_n(x)\}$ where $f_n(x) = \frac{n}{x+n}, x \geq 0$ is uniformly convergent on $[0, m]$

21. Define Lipschitz function with suitable examples. Does the function $f(x) = x^3$ satisfy the Lipschitz condition in the interval $[0, 2]$
22. Define the convergence of Improper Integral of first kind and Investigate the convergence of $\int_1^\infty e^{-x^2} dx$
23. Prove that if $\int_a^\infty |f(x)| dx$ converges, then $\int_a^\infty f(x) dx$ converges.
24. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$
25. Prove that $\beta(m, n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy$
26. Evaluate $\int_0^{\frac{\pi}{2}} \sin^{10}\theta \, d\theta$

(10 x 4 = 40 Marks)

Section C

Answer any ***six*** questions. Each question carries 7 marks.

27. State and Prove Maximum - Minimum theorem for continuous functions.
28. Let I be an interval and let $f: I \longrightarrow \mathbb{R}$ be continuous on I. Then prove that $f(I)$ is an interval.
29. State and prove any two properties of Riemann Integral.
30. Prove that an unbounded function cannot be Riemann Integrable.
31. State and prove Squeeze theorem of Riemann Integrable functions.
32. Show that $\{f_n(x)\}$ where $f_n(x) = \frac{x}{nx+1}$ converges uniformly on $[0, \infty)$
33. Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \longrightarrow 0$
34. State the Integral test for the convergence of a series of functions. Using Integral test, Evaluate the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2}$
35. Establish the relation between Beta function and Gamma function.

(6 x 7 = 42 Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. i) State and prove Location of Roots theorem.
    ii) Prove that every odd degree real polynomial has at least one real root.
37. i) Define Indefinite integral of f where $f \in \mathcal{R}[a, b]$ with base point a
    ii) State and prove the Fundamental theorem of Calculus (Second form).
38. i) Find the value of $\int_2^3 (x-2)^2(3-x)^3 dx$
    ii) Prove that $\beta(p, q) = \beta(p+1, q) + \beta(p, q+1)$

(2 x 13 = 26 Marks)

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