

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19U MTS1 C01 - MATHEMATICS - 1

(Mathematics - Complementary Course)

(2019 Admission onwards)

Time : 2.00 Hours

Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)Answer *all* question. Each question carries 2 marks.

1. Find $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 + 2x - 8}$
2. Differentiate $x^{95} + x^{23} + 2x^2 + 4x + 1$ with respect to x .
3. Find $\frac{d}{dx} \left(\frac{1}{(x^2 + 9)^2} \right)$
4. Differentiate $\sqrt{x^3 - 5}$
5. If $x^2 + y^2 = 1$, express $\frac{dy}{dx}$ in terms of x and y .
6. Find $\int \left(\frac{t^3 - 8t + 1}{t^{3/2}} \right) dt$
7. Let $f(x)$ be the step function defined by $f(x) = \begin{cases} -1 & \text{if } x < 0, \\ -2 & \text{if } x \geq 0, \end{cases}$ Show that f is discontinuous at x_0 .
8. Find and classify the critical points of the function $f(x) = x^3 + 3x^2 - 6x$ by using first derivative test.
9. Use the second derivative test to analyze the critical points of the function $f(x) = x^4 - x^2$
10. Let $f(x) = x^7 - x^5 - x^4 + 2x + 1$. Prove that somewhere between -1 and 1 the tangent line to the graph of f has slope 2 .
11. An object on the x axis has velocity $v = 2t - t^2$ at time t . If it starts out at $x = -1$ at time $t = 0$, where is it at time $t = 3$? How far has it travelled?

12. Let $F(x) = \int_2^x \frac{1}{1 + s^2 + s^3} ds$. Find $F'(3)$.

(Ceiling: 20 Marks)

Part B (Short essay questions)

Answer **all** question. Each question carries 5 marks.

13. Use the formal definition find the derivative of $f(x) = \frac{1}{x}$ at $x = 1$

14. Using linear approximation calculate an approximate value for $\frac{2}{\sqrt{0.99 + (0.99)^2}}$

15. Find the equation of the line tangent to the parametric curve $x = t^2 + 1, y = \frac{1}{t^4 + 1}$ at $t = 2$.

16. Find the critical points, endpoints, maximum and minimum points and values of the function $f(x) = x^4 - 4x^2 + 7$ on the interval $[-4, 2]$

17. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin 3x - 3x}{x^3} \right)$

18. Compute the exact value of $\int_0^1 (x^5) dx$ by using Reimann sums and the formula

$$1^5 + 2^5 + 3^5 + \dots + N^5 = \frac{N^6}{6} + \frac{N^5}{2} + \frac{5N^4}{12} - \frac{N^2}{12}$$

19. The curves $x = y^2$ and $x = 1 + \frac{1}{2}y^2$ divide the xy plane into five regions only one of which is bounded. Find the area of this bounded region.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any **one** question. Each question carries 10 marks.

20. a) Suppose that it costs $\frac{30x + 0.04x^2}{1 + 0.0003x^3}$ dollars if x calculators are made, where $0 \leq x \leq 100$, and that calculators are priced at $100 - 0.05x$ dollars. If all x calculators are sold, what is the marginal profit?

b) Compute the second derivative $\frac{d^2}{dx^2} \left(\frac{x^3 - 1}{x^4 + 8} \right)$

21. The region between the graphs of $\sin x$ and x on $\left[0, \frac{\pi}{2}\right]$ is revolved about the x axis. Find the volume of the solid thus obtained.

(1 × 10 = 10 Marks)
