19U313S (Pages: 2) Name: …………………

Reg. No…………………..

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020
(CUCBCSS - UG)
CC15U MAT3 B03 / CC18U MAT3 B03 - CALCULUS AND ANALYTIC GEOMETRY
(Mathematics - Core Course)
(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours   Maximum: 80 Marks

PART-A (Objective Type)
Answer *all* questions. Each question carries 1 mark.

1. Evaluate $\int \tan x \, dx$.

2. If $\cosh x = \frac{5}{4}$, then $\sinh x = \cdots$.

3. Evaluate $\lim\limits_{x \to 0^+} \left(\frac{\sin x}{x^2} \right)$

4. Give an example of a bounded sequence which is not convergent.

5. Evaluate $\lim\limits_{n \to \infty} \left(\frac{\ln n}{n} \right)$

6. Find the sum of the series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

7. Give an example of an alternating convergent series.

8. Write the vertex of the parabola $(x - 2)^2 = -4(y - 3)$.

9. Find the slope of the curve $x = 4 \sin t$ and $y = 2 \cos t$ at $t = \pi/4$

10. Replace the Cartesian equation $x^2 + y^2 = 4$ by equivalent polar equation.

11. Write the polar equation of a circle with centre at $(2,0)$ and passing through the origin.

12. Write the standard polar equation of the ellipse.

(12 x 1 =12 Marks)

PART-B (Short Answer Type)
Answer any *nine* questions. Each question carries 2 marks.

13. Evaluate $\int_1^2 \frac{2^{\ln x}}{x} dx$.

14. Prove that $\operatorname{csch}^{-1} x = \sinh^{-1}\left(\frac{1}{x}\right)$.

15. Evaluate $\lim\limits_{x \to 0^+} \left(\frac{\ln x}{\cot x} \right)$.

16. Show that the function 4^x grows faster than 3^x.

17. For any $x > 0$, prove that $\lim\limits_{n \to \infty} (x)^{1/n} = 1$.

18. Find the limit of the convergent sequence $\{a_n\}$, where $a_n = \left(\frac{1}{n}\right)^{1/\ln n}$.

19. Investigate the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n^3+1}{2^n+1}\right)$.

20. Find the eccentricity of the ellipse $2x^2 + 3y^2 = 6$.

21. Graph the set of points whose polar coordinates satisfy the conditions $0 \leq \theta \leq \frac{\pi}{2}$, $1 \leq r \leq 2$.

22. Find the area of the Lemniscates of Bernoulli $r^2 = \cos 2\theta$.

23. Polar equation of a conic is $r = \frac{16}{4+4\sin\theta}$. Identify the conic and find its directrix.

24. Show that the point $\left(\frac{1}{2}, \frac{3\pi}{2}\right)$ lies on the curve $r = -\sin\left(\frac{\theta}{2}\right)$

(9 x 2 = 18 Marks)

PART-C (Short Essay Type)
Answer any *six* questions. Each question carries 5 marks.

25. Express $\tanh^{-1} x$ in terms of logarithms.

26. Show that $\sqrt{x^2 + 5}$ and $\left(2\sqrt{x} - 1\right)^2$ grow at the same rate as $x \to \infty$

27. Find $\lim\limits_{n\to\infty} \left(1 + \frac{x}{n}\right)^n$.

28. Examine the convergence of the series $\frac{1}{2} + \frac{1}{2^2} + \frac{3}{2^3} + \frac{1}{2^4} + \frac{5}{2^5} + \frac{1}{2^6} + \frac{7}{2^7} + \cdots$.

29. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$ converges conditionally.

30. Find the Taylor series generated by $f(x) = e^x$ at $x = 1$. Where if anywhere, does the series converge?

31. Find the centre, eccentricity, foci and directories of the ellipse
$9x^2 + 25y^2 - 18x - 100y - 116 = 0$.

32. Find the area of the surface of the solid formed by the revolution of the cardioid
$r = 1 + \cos\theta$ about the $x - axis$.

33. Find the points of intersection of the curves $r^2 = 4\cos\theta$ and $r = 1 - \cos\theta$.

(6 x 5 =30 Marks)

PART - D (Essay Type)
Answer any *two* questions. Each question carries 10 marks.

34. Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$, where $p > 0$

35. Find Maclaurin series for $f(x) = \ln(1 + x)$. Also find Taylor polynomials of orders 0, 1, 2 and 3 generated by f at zero.

36. Draw the circles $r = a\sqrt{2}$ and $r = 2a\cos\theta$ and find the area shared by the circles.

(2 x 10 = 20 Marks)

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