19U314S (Pages: 2) Name……………………

Reg. No…………………

THIRD SEMESTER B.Sc. DEGREE EXAMINATION NOVEMBER 2020

(CUCBCSS-UG)

CC15U MAT3 C03 / CC18U MAT3 C03 - MATHEMATICS

(Mathematics - Complementary)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum:80 Marks

PART A

Answer ***all*** questions. Each question carries 1 mark.

1. Determine c if y = c$e^{-x^2}$and y (0) = 0.5.
2. Degree of the differential equation $(y')^2 + y = x^2 - 2$ is ……
3. Solve $y' = 2y$.
4. Is the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$ singular?
5. The eigen values of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ are …………..
6. State Cayley Hamilton theorem.
7. What is the volume of the parallelepiped with co-edge vectors $\bar{a}$, $\bar{b}$ & $\bar{c}$.
8. $(\vec{\imath}, \vec{\jmath}, \vec{k}) =$ …..
9. Find $\nabla\varphi$ if φ = xy^2z.
10. The directional derivative of a scalar field φ at a point in the direction of a unit vector $\bar{a}$ is …………..
11. Give an example of a non-orientable surface.
12. State Divergence theorem.

(12 x 1 = 12 Marks)

PART B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Find the general solution $y' = 1 + 0.01\, y^2$
14. Solve $y' - y = e^{2x}$
15. Find the integrating factor of the differential equation $\frac{dy}{dx} + y \tan x = cos^3 x$.
16. If 1 and 3 are the eigen values of A find the eigen values of A^{-1}
17. Reduce to normal form and hence find the rank of the matrix $\begin{pmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{pmatrix}$
18. Find unit normal to the surface $x^2y + 2xz = 4$ at (1,0,2)
19. Prove that div $\bar{r} = 3$
20. Find the parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$
21. Find the angle between the lines $3x + 5y = 0$ and $4x - 2y = 1$ using vector method.

22. Determine the arc length of the curve $x = \cos t\,, y = \sin t$ between $t = 0$ and $t = 2\pi$

23. Check for the path independence: $\int_C\ 3z^2dx + 6xzdx$

24. Show that $yz\vec{\imath} + zx\vec{\jmath} + xy\vec{k}$ is irrotational.

(9 x 2 = 18 Marks)

PART C

Answer any ***six*** questions. Each question carries 5 marks.

25. Show that $(2xy + y - tany)\,dx + (x^2 - xtan^2y + sec^2y + 2)dy = 0$ is exact and solve it.

26. Solve $y' - Ay = -By^2, A, B$ are positive.

27. Find the rank of the matrix A, by reducing to Echelon form $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

28. Using Cayley Hamilton theorem find A^{-1}, if A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

29. Find the tangential and normal acceleration of a body moving along a path with position vector $\bar{r}(t) = cost\bar{\imath} + sin2t\bar{\jmath} + cos2t\bar{k}$

30. If $\nabla\varphi = (y + y^2 + z^2)\bar{\imath} + (x + z + 2xy)\,\bar{\jmath} + (y + 2zx)\,\bar{k}$, find the potential function φ such that $\varphi(1,1,1) = 3$

31. Prove that $div(curl\ \bar{v}) = 0$

32. If $\bar{F} = (3x^2 + 6y)\bar{\imath} - 14yz\bar{\jmath} + 20xz^2\bar{k}$, evaluate $\int_C \bar{F}.d\bar{r}$, from (0,0,0) to (1,1,1) along the paths.

    (i) $x = t;\ y = t^2;\ z = t^3$

    (ii) The straight line joining (0,0,0) and (1,1,1)

33. Using Green's theorem evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(6 x 5 = 30 Marks)

PART D

Answer any ***two*** questions. Each question carries 10 marks.

34. Find the orthogonal trajectories of the family of circles $x^2 + (y - c)^2 = c^2$

35. Solve completely the system of equations

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0$$

36. Verify Stokes theorem for $\bar{F} = [y, z, x]$, S is the paraboloid $z = f(x, y) = 1 - (x^2 + y^2)$, $z \geq 0$ and C is its boundary.

(2 x 10 = 20 Marks)

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