19U301 (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS - UG)

CC19U MTS3 B03 - CALCULUS OF SINGLE VARIABLE - II

(Mathematics - Core Course)

(2019 Admission - Regular)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Find the derivative of $f(x) = \ln(2x^2 + 1)$
2. Define the logarithmic function $f(x) = \log_a(x)$, where $a > 0$ and $a \neq 1$. What are its domain and range?
3. Find the derivative of $g(x) = \tanh(1 - 3x)$.
4. State L Hopital's Rule.
5. Find an expression for the n^{th} term of the sequence $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \cdots\right\}$
6. Find the n^{th} partial sum of $\sum_{n=1}^{\infty} \frac{4}{(2n+3)(2n+5)}$
7. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges or diverges.
8. State root test for series.
9. Find a rectangular equation whose graph contains the curve C with parametric equations $x = \cosh t, y = \sinh t$.
10. Find the points on the curve $x = 2t^2 - 1, y = t^3$ at which the slope of the tangent line is $m = 3$.
11. The point $(2\sqrt{3}, -2)$ is given in rectangular coordinates. Find its representation in polar coordinates.
12. Find parametric equations for the line passing through the point $(1, 3, 2)$ and parallel to the vector $\mathbf{v} = \langle 2, 4, 5\rangle$.
13. The point $(2, 0, 3)$ is expressed in rectangular coordinates. Find its cylindrical coordinates.
14. Find $\lim\limits_{t\to 2} \left[\sqrt{t}\,\bar{i} + \left(\frac{t^2-4}{t-2}\right)\bar{j} + \frac{t}{t^2+1}\bar{k}\right]$
15. Find unit tangent vector $\overline{T}(t)$ of $\bar{\gamma}(t) = 2\sin 2t\,\bar{i} + 2\cos 2t\,\bar{j} + 3\bar{k}$ at $t = \frac{\pi}{6}$

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Find the inverse of the function defined by $f(x) = \frac{1}{\sqrt{2x-3}}$

17. Find the derivative of $f(x) = \frac{2^x}{\sqrt{3^x+1}}$

18. Find the value of p for which $\int_1^\infty \frac{1}{x^p} dx$ is convergent.

19. Use limit comparison test to determine whether the series $\sum_{n=2}^{\infty} \frac{n}{n^2+1}$ is convergent or not.

20. Find the radius of convergence and the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{\sqrt{n}}$

21. Find all the points of intersection of the curves $r = 1$ and $r = 1 + \cos\theta$

22. Sketch the surface represented by the equation $4x^2 + y^2 + z^2 = 4$

23. Find the velocity and position vector of an object with acceleration $\overline{a}(t) = 6t\overline{i} + \overline{j} + 2\overline{k}$ and initial position and initial velocity given by $\overline{\gamma}(0) = \overline{i} + 2\overline{j} + \overline{k}$ and $\overline{v}(0) = \overline{i} + 2\overline{k}$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Use implicit differentiation to find $\frac{dy}{dx}$ for $x \ln y + e^{-x} - ye^y = 0$.

25. a) Find an approximation of the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ accurate to two decimal places.

    b) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n+2}$ converges or diverges.

26. Find the Taylor series for $f(x) = \ln x$ at 1, and determine its interval of convergence.

27. Find the arc length function $S(t)$ for the circle C in the plane described by $\overline{\gamma}(t) = 2\cos t\overline{i} + 2\sin t\overline{j}, \quad 0 \le t \le 2\pi$. Then find a parametrization of C in terms of S.

(2 × 10 = 20 Marks)

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