19U300 (Pages: 2) Name: .....

Reg.No: .....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS - UG)

CC19U MTS3 C03 - MATHEMATICS - III

(Mathematics - Complementary Course)

(2019 Admission - Regular)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Find the parametric equation of the tangent line to the curve $x = t, y = \frac{1}{2}t^2, z = \frac{1}{3}t^3$ at $t = 2$.
2. Describe the level surfaces of the function $f(x, y, z) = \dfrac{x^2}{9} + \dfrac{z^2}{4}$
3. Find the gradient of the function $F(x, y, z) = xy\cos yz$
4. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, prove that $curl \quad \vec{r} = \vec{0}$
5. Evaluate $\int_c 2xydx$ on the curve $x = 5\cos t, y = 5\sin t, \quad 0 \leq t \leq \frac{\pi}{4}$
6. Determine whether the vector field $\mathbf{F}(x, y) = -ye^{-xy}\mathbf{i} - xe^{-xy}\mathbf{j}$ is conservative.
7. Find the Jacobian of the transformation $x = r\cos\theta, y = r\sin\theta$.
8. Define a region in the complex plane.
9. Show that the Cauchy Riemann equations are satisfied for $f(z) = z^3$ at every point.
10. Find the value of $ln(-1 - i)$
11. Find the value of $\oint_C \frac{(z^2-9)}{\cosh z}dz$, where C is the unit circle $|z| = 1$
12. Evaluate $\oint_C \frac{1+2e^z}{z}dz$ where C is $|z| = 1$

(Ceiling: 20 Marks)

Part B (Short essay questions)

Answer ***all*** questions. Each question carries 5 marks.

13. If $\mathbf{r}(t) = \mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$ gives the position vector of a moving particle. Find the tangential and normal components of acceleration at any time t.

14. Find an equation of the tangent plane to the graph of $x^2 - 4y^2 + z^2 = 16$ at $(2, 1, 4)$.

15. Evaluate $\iint_R xe^{y^2} dA$ over the region R in the first quadrant bounded by the graphs of $y = x^2, x = 0$ and $y = 4$.

16. Consider the surface of the sphere $x^2 + y^2 + z^2 = a^2; a > 0$. Find two orientations of the surface.

17. Convert the point $(-5, -5, 0)$ given in rectangular coordinates to spherical coordinates.

18. Express the complex number $3i^5 - 2i^3 - 3i^2 + 5i - 9$ in the form $a + ib$.

19. Compute the circulation and net flux for the function $f(z) = \overline{z}$ over the square C with vertices $z = 0, z = 1, z = 1 + i, z = i$.

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. Each question carries 10 marks.

20. Verify Green's theorem by evaluating both the integrals, $\oint_C ((x - y)dx + xydy) = \iint_R (y + 1)dA$ where C is the triangle with vertices $(0, 0), (1, 0), (1, 3)$ taken in anticlockwise direction.

21. a) Evaluate $\int_\pi^i e^z \cos z dz$
    b) Evaluate $\int_i^{i+1} ze^z dz$

(1 × 10 = 10 Marks)

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