19P306S (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCSS-PG)

(Supplementary/Improvement)

CC18P MT3 C13 – COMPLEX ANALYSIS

(Mathematics)

(2018 Admission)

Time: Three Hours Maximum : 36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Find the point of **S** corresponding to the point $1+i \in \mathbb{C}$ in the stereographic projection.
2. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a^n z^n$, $a \in \mathbb{C}$.
3. Show that a Mobius transformation T satisfies $T(0)=\infty$ and $T(\infty)=0$ iff $Tz=\frac{a}{z}$ for some $a \in \mathbb{C}$.
4. Let γ be a rectifiable curve and f is a function continuous on $\{\gamma\}$, then prove that $\int_{-\gamma} f = -\int_{\gamma} f$
5. If $\gamma:[0,1] \to \mathbb{C}$ is a closed rectifiable curve and $a \notin \{\gamma\}$. Show that $\frac{1}{2\pi i}\int_{\gamma} \frac{dz}{z-a}$ is an integer.
6. Show that a bounded entire function is a constant.
7. Evaluate $\int_{\gamma} \frac{z^2-1}{z^2+1} dz$ where γ is the circle $|z-i|=1$ in the clockwise direction around the circle.
8. State and prove independence of path theorem.
9. When two rectifiable curves are said to be FEP homotopic?
10. Determine the nature of the singularity of $f(z)=\frac{\cos z-1}{z}$ at $z=0$.
11. Find the residue at the singular points of the function $f(z)=\frac{2z}{(z+4)(z-1)^2}$
12. Define the term meromorphic functions. Give example.
13. If $|a|<1$ and φ_a is a Mobius transformation defined by $\varphi_a(z)=\frac{z-a}{1-\bar{a}z}$. Show that $(\varphi_a)^{-1}=\varphi_{-a}$
14. State and prove the second version of maximum modulus principle.

($14 \times 1 = 14$ Weightage)

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Let $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ has a radius of convergence $R > 0$. Prove that f is infinitely differentiable on $B(a; R)$.
16. Show that $u(x, y) = 4xy - x^3 + 3xy^2$ is harmonic. Also find its harmonic conjugate.
17. State and prove symmetry principle.
18. If γ is piecewise smooth and $f: [a, b] \to \mathbb{C}$ is continuous then prove that

$$\int_a^b f d\gamma = \int_a^b f(t)\gamma'(t)\, dt$$

19. Evaluate $\int_\gamma \frac{z^2+1}{z(z^2+4)} dz$, where $\gamma(t) = \mathrm{r}e^{it}$, $0 \le t \le 2\pi$ for all possible values of r such that $0 < r < 2$ and $2 < r < \infty$.
20. If G is simply connected and $f: G \to \mathbb{C}$ is analytic in G. show that f has a primitive in G.
21. Let G be an open set and let $f: G \to \mathbb{C}$ be a differentiable function, then prove that f is analytic on G.
22. State and prove residue theorem.
23. Evaluate $\int_0^\pi \frac{1}{(a+\cos\theta)^2} \mathrm{d}\theta$ where $a > 1$.
24. State and prove Rouche's theorem.

$(7 \times 2 = 14$ Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. If z_1, z_2, z_3, z_4 be four distinct points in $\mathbb{C}_\infty$. Prove that (z_1, z_2, z_3, z_4) is a real number iff all four points lie on a circle.
26. If $f: G \to \mathbb{C}$ is analytic and $\bar{B}(a; r) \subset$ G. Then show that $f^{(n)}(a) = \frac{n!}{2\pi i}\int_\gamma \frac{f(w)}{(w-a)^{n+1}} dw$, where $\gamma(t) = \mathrm{a} + \mathrm{r}e^{it}$, $0 \le t \le 2\pi$. Using this result evaluate $\int_\gamma \frac{e^z - e^{-z}}{z^n} dz$ where n is a positive integer and $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$
27. State and prove Laurent series development theorem.
28. Show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$.

$(2 \times 4 = 8$ Weightage)

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