19P302 (Pages: 2) Name: ....

Reg. No.: ....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCSS-PG)

CC19P MTH3 C12 - COMPLEX ANALYSIS

(Mathematics)

(2019 Admission Regular)

Time: Three Hours Maximum : 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Suppose $S = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$ and $S_T : \mathbb{R}^2 \to S \setminus \{\mathbb{N}\}$ is stereographic projection such that $T(z) = (x_1, x_2, x_3)$ where $z = x + iy$, express x_1, x_2, x_3 in terms of x and y. Also, find the corresponding images of $0, 1+i$ and $3+2i$.
2. By expressing $\cos z$ and $\sin z$ in terms of exponential function, show that $\cos^2 z + \sin^2 z = 1$.
3. Show that a Möbius transformation can have atmost two fixed points unless it is the identity function.
4. If $\gamma : [0, 2\pi] \to \mathbb{C}$ by $\gamma(t) = e^{int}$ where $n \in \mathbb{Z}$, find $\displaystyle\int_\gamma \frac{1}{z} dz$.
5. State and prove Open Mapping Theorem.
6. Evaluate $\displaystyle\int_\gamma \frac{2z+1}{z^2+z+1} dz$.
7. Justify that $\dfrac{\sin z}{z}$ has a removable singularity and it is isolated.
8. State Hadamard's three circle theorem.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. If $\sum a_n(z-a)^n$ is a given power series with radius of convergence R, then prove that $R = \lim \left|\dfrac{a_n}{a_{n+1}}\right|$, if this limit exists.
10. Define the cross ratio of z_1, z_2, z_3 and z_4. Prove that if T is a Möbius transformation and z_2, z_3 and z_4 are distinct points, then $(z_1, z_2, z_3, z_4) = (T(z_1), T(z_2), T(z_3), T(z_4))$.

11. If γ is a piecewise smooth and $f : [a, b] \to \mathbb{C}$ is continuous then, show that $\int_a^b f d\gamma = \int_a^b f(t)\gamma'(t)dt$.

UNIT II

12. Evaluate $\int_\gamma \frac{z}{(z-1)^2} dz$ where $\gamma(t) = 1 + e^{it}$ and $0 \le t \le 2\pi$.

13. Suppose γ is a closed rectifiable curve in $\mathbb{C}$, then define the index of γ with respect to a point $a \notin \gamma$. Can it be generally a real number?

14. State and prove Independence of path theorem.

UNIT III

15. Calculate the integral: $\int_0^\pi \frac{d\theta}{(a + \cos\theta)^2}$ where $a > 0$.

16. State and prove Argument Principle.

17. State and prove Schwarz lemma.

(6 × 2 = 12 Weightage)

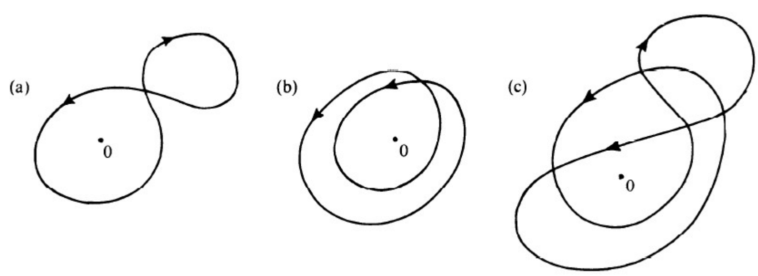
PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let G be either the whole plane or $\mathbb{C}$ or some open disk, if $u : G \to \mathbb{R}$ is a harmonic function, then prove that u has a harmonic conjugate.

19. If z_1, z_2, z_3 and z_4 are four distinct points in $\mathbb{C}$, then prove (z_1, z_2, z_3, z_4) is a real number if and only if all four points lie on a circle.

20. Evaluate $\int_\gamma \frac{e^z - e^{-z}}{z^4} dz$ where γ is one of the curves depicted below. Also justify for each cas



21. Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{1 + x^4} dx$.

(2 × 5 = 10 Weightage)

****