19P304 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS-PG)

CC19P MTH3 C14 - PDE AND INTEGRAL EQUATIONS

(Mathematics)

(2019 Admission Regular)

Time: Three Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Find the solution of $u_x + u_y = 1$ subject to the initial condition $u(x,0) = f(x)$
2. Define eikonal equation.
3. Find the canonical form of $u_{xx} + xu_{yy} = 0,\ x > 0$
4. Find a canonical transformation $q = q(x,y), r = r(x,y)$ and the corresponding canonical form for the equation $u_{xx} + xu_{yy} = 0,\ x > 0$
5. The Cauchy problem for nonhomogeneous wave equation admits at most one solution. justify
6. Prove that, Let v be a function in C_H satisfying $v_t - k\Delta v < 0$ in Q_T. Then v has no local maximum in Q_T. Moreover, v attains its maximum in ∂pQ_T
7. If $y''(x) = F(x)$ and y satisfies the end condition $y(0) = 0$, $y(1) = 0$. Show that $y(x) = \int_0^x (x - \xi)F(\xi)d\xi - x\int_0^1 (1 - \xi)F(\xi)d\xi$
8. Define separable kernel with an example.

(8 × 1 = 8 Weightage)

Part B

Answer any ***six*** questions. Each question carries 2 weightage.

Unit I

9. Solve the equation $xu_x + (x + y)u_y = 1$ with the initial conditions $u(1,y) = y$
10. Write the eikonal equation in the form $F(x,y,u,p,q) = P^2 + q^2 - n_0^2 = 0$ with initial conditions are $x(0,s) = s, y(0,s) = 2s, u(0,s) = 1$
11. Find a coordinate system (s,t) in which the equation $u_{xx} - 6u_{xy} + 9u_{yy} = xy^2$ has the form $9v_{tt} = \frac{1}{3}(s - t)t^2$

Unit II

12. State and prove the mean value principle.
13. State and prove the strong maximum principle.

14. Solve the heat equation $u_t = 12u_{xx}, 0 < x < \pi,\ t > 0$ subject to the boundary and initial conditions.

$$u_x(0,t) = u_x(\pi,t) = 0, \qquad t \geq 0$$
$$u(x,0) = 1 + sin^3 x, \qquad 0 \leq x \leq \pi$$

Unit III

15. Determine the characteristic value of λ for the equation $y(x) = F(x) + \lambda \int_0^{2\pi} \cos(x+\xi)\, d\xi$

16. Solve by iterative procedure $y(x) = \lambda \int_0^1 x\xi y(\xi) d\xi + 1$

17. Prove that characteristic numbers of Fredholm equation with real symmetric kernel are all real.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (a) Show that there exists a unique solution for the system $u_x = 3x^2y + y$

$u_y = x^3 + x$ together with the initial condition $u(0,0) = 0$

(b) Solve the equation $-yu_x + xu_y = u$ subject to the initial condition $u(x,0) = \psi(x)$

19. Find a compatibility condition for the Cauchy problem

$u_x^2 + u_y^2 = 1,\ u(\cos s, \sin s) = 0,\ 0 \leq s \leq 2\pi$

20. (a) State and Prove Greens identity.

(b) Using the separation of variables method find a solution of a vibrating string with fixed ends.

$$u_{tt} = u_{xx}, \quad 0<x<\pi,\ t>0$$
$$u(0,t) = u(\pi,t) = 0, \qquad t \geq 0$$
$$u(x,0) = sin^3 x,\ \ 0 \leq x \leq \pi$$
$$u_t(x,0) = \sin 2x,\ \ 0 \leq x \leq \pi$$

21. (a) Find the Greens function associated with the problem

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \ (\lambda x^2 - 1)y = 0,\ \ y(0) = 0, \qquad y(1) = 0$$

(b) Transform the problem $y'' + y = x, y(0) = 1, y'(1) = 0$ into a Fredholm Integral equation.

(2 × 5 = 10 Weightage)

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