19P362 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCSS-PG)

CC19P ST3 C12 - STOCHASTIC PROCESSES

(Statistics)

(2019 Admission Regular)

Time : Three Hours Maximum : 30 Weightage

PART A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Describe classification of the stochastic processes with suitable examples.
2. Define recurrent and transient states with examples.
3. What is (i) non homogeneous Poisson process and (ii) compound Poisson processes
4. Describe Birth and Death model.
5. Describe (i) Renewal reward process (ii) Regenerative process.
6. What do you mean by queue? Briefly explain Kendall's Notation.
7. Describe Brownian motion.

(4 x 2 = 8 Weightage)

PART B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Show that any Markov chain is completely described by one step transition probabilities and the initial distribution.
9. Check whether the following Markov chain with four states 0, 1, 2, and 3 having TPM given below is ergodic

$$\begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

10. If $\{N(t)\}$ is a Poisson process, derive auto-correlation between $N(t)$ and $N(t+s)$, t, $s > 0$
11. If E(R) < ∞ and E(X) < ∞ Then show that:
    (i) with probability 1, $\lim_{t\to\infty} \frac{R(t)}{t} = \frac{E(R)}{E(X)}$
    (ii) $\lim_{t\to\infty} \frac{E(R(t))}{t} = \frac{E(R)}{E(X)}$
12. Describe insurance ruin problem.
13. For an M|M|1 queueing model, find the expected number of customers in the system and the queue

14. What are network of queues? Illustrate with a suitable example.

(4 x 3 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

15. a) For a continuous time Branching process, obtain mean and variance

    b) Show that (i) $P_n(s) = P_{n-1}(P(s))$ (ii) $P_n(s) = P(P_{n-1}(s))$

16. Let $\{N(t), t \geq 0\}$ and $\{M(t), t \geq 0\}$ be independent non homogeneous Poisson processes with respective intensity functions $\lambda(t)$ and $\mu(t)$ and let $N^*(t) = N(t) + M(t)$. Then show that the following are true

    a) $\{N^*(t), t \geq 0\}$ is a non homogeneous Poisson process with intensity functions $\lambda(t) + \mu(t)$

    b) Given that an event of the $\{N^*(t)\}$ process occurs at time t, then independent of what occurred prior to t , the event t was from the $\{N(t)\}$ process with probability $\frac{\lambda(t)}{\lambda(t)+\mu(t)}$

17. For a renewal process $\{N(t), t \geq 0)$, show that

    a) With probability 1, $\frac{N(t)}{t} \to \frac{1}{\mu}$ as $t \to \infty$.

    b) $\frac{m(t)}{t} \to \frac{1}{\mu}$ as $t \to \infty$

18. Describe $M|G|1$ queue

(2 x 5 = 10 Weightage)

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