19P203S (Pages: 2) Name. ....

Reg. No. ....

SECOND SEMESTER M.Sc. DEGREE EXTERNEL EXAMINATION, APRIL 2020

(CUCSS - PG)

CC18P MT2 C08 – REAL ANALYSIS II

(Mathematics)

(2018 Admission - Supplementary/Improvement)

Time: Three Hours Maximum: 36 weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Show that the outer measure of a singleton set is zero.
2. Show that $m^*(A \cup B) = m^*(B)$if $m^*(A) = 0$.
3. Compute $\int \chi_A$ where A is a measurable set.
4. Show that $essential\, sup f = -ess$ential $inf(-f)$
5. Find the integral of Dirichlet function on [0,1].
6. Show that $f = 0$ $a.e$ need not imply $f = 0$.
7. Define Lebesgue set. Find the lebesgue set of $f(x) = x^2$.
8. If f is of bounded variation on $[a, b]$ then prove that f is bounded on $[a, b]$
9. Show that a signed measure υ is $\sigma -$ finite if and only if $|\upsilon|$ is $\sigma -$ finite.
10. Show that $f(x) = x$ is absolutely continuous on $[a, b]$
11. Let A is a positive set with respect to a signed measure υ on a measurable space $[X, \mathbb{S}]$. For $\in \mathbb{S}$, let $\mu(E) = \upsilon(E \cap A)$. Prove that μ is a measure on $[X, \mathbb{S}]$.
12. State Riesz representation theorem for $C(I)$.
13. Let R be a ring and let μ a measure on R. If $A, B \in R$with $A \subseteq B$, then prove that $\mu(A) \leq \mu(B)$.
14. Let f and g be absolutely continuous on the finite interval $[a, b]$, prove that fg is absolutely continuous on $[a, b]$

($14 \times 1 = 14$ Weightage)

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Show that the collection of measurable subsets of $\mathbb{R}$ is a σ- algebra.
16. Show that if E is measurable then $\forall \varepsilon > 0, \exists$ an open set O such that $E \subseteq O$ and $m^*(O \setminus E) \leq \varepsilon$
17. Let E be a measurable set. Then show that for every $y \in \mathbb{R}$, $E + y = \{x + y : x \in E\}$ is measurable and $m(E + y) = m(y)$
18. State and prove monotone convergence theorem
19. Prove that if f be a non negative measurable function then$\int f = 0$ if and only if $f = 0$ $a.e$

20. State and prove Jordan’s Theorem.

21. Show that a monotone function is of bounded variation

22. Let $f \in L(a, b)$ with indefinite integral F then prove that $F' = f$ $a.e$ on $[a, b]$

23. If μ and υ are measures such that $\upsilon \ll \mu$ and $\upsilon \perp \mu$ then show that υ is identically zero.

24. Show that if μ^* be an outer measure on $\mathcal{H}(R)$ and let S^* denote the class of μ^* measurable sets. Then S^* is a σ- ring

(7 × 2 = 14 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Show that the outer measure of an interval is its length.
26. Let f be an increasing real valued function on [a, b], prove that f is differentiable almost every where, and the derivative f ' is measurable.
27. State and prove Vitali Convergence Theorem.
28. State and prove Fatou's lemma.

(2× 4 = 8 weightage)

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