19P205S (Pages: 2) Name ……..………………

Reg. No…..………………..

SECOND SEMESTER M.Sc. EXTERNAL EXAMINATION, APRIL 2020

(CUCSS - PG)

CC17P MT2 C10 - ODE AND CALCULUS OF VARIATIONS

(Mathematics)

(2017, 2018 Admissions - Supplementary/Improvement)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define the radius of convergence of the power series $\sum a_n x^n$
2. Determine the nature of the point $x = \infty$ for Bessel's equation
$$x^2y'' + xy' + (x^2 - p^2)y = 0$$
3. Verify that $P_{2n}(0) = (-1)^n \frac{1.3.5......(2n-1)}{2^n n!}$ where P_n(x) is the n^{th} degree Legender polynomial.
4. Show that $\lim_{b\to\infty} F\left(a, b, a, \frac{x}{b}\right) = e^x$
5. Describe the phase portrait of the system $\frac{dx}{dt} = 1$, $\frac{dy}{dt} = 2$
6. Show that $\frac{d}{dx}[x\, J_1(x)] = x\, J_0(x)$.
7. Find the indicial equation and its roots of the equation
$$x^3y'' + (cos2x - 1)y' + 2xy = 0.$$
8. Find the critical points of the non linear system $\frac{dx}{dt} = y(x^2 + 1)$, $\frac{dy}{dt} = 2xy^2$.
9. Determine whether the function $2x^2 - 3xy + 3y^2$ is positive definite, negative definite or neither.
10. Show that $f(x, y) = xy^2$ satisfies Lipschitz condition on any rectangle a $\leq x \leq b$ and $c \leq y \leq d$.
11. What is the isoperimetric problem?
12. Find the normal form of the Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$,where p is a non –negative constant.
13. Find the extremals for the integral $\int_{x_1}^{x_2} \frac{\sqrt{1+(y')^2}}{y}$ dx .
14. Show that every non-trivial solution of $y'' + (sin^2x + 1)y = 0$ has an infinite number of positive zeros.

(14 x 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Express $sin^{-1}x$ in the form of a power series $\sum a_n x^n$ solving $y' = (1-x^2)^{-1/2}$, in two ways. Hence Obtain the formula $\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{3.2^3} + \frac{1.3}{2.4}\cdot\frac{1}{5.2^5} + \frac{1.3.5}{2.4.6}\cdot\frac{1}{7.2^7} + \ldots\ldots\ldots\ldots$

16. Find the general solution of the equation $x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ near the singular point $x = 0$.

17. Show that the Legendre polynomial $P_n(x)$ satisfies the orthogonality property

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$$

18. Show that $\frac{2P}{x}J_P(x) = J_{P-1}(x) + J_{P+1}(x)$.

19. Verify that (0,0) is a simple critical point of the system $\frac{dx}{dt} = 4x - 3y$, $\frac{dy}{dt} = 8x - 6y$ and determine its nature and stability properties.

20. Show that (0,0) is an asymptotically stable critical point for the system

$$\frac{dx}{dt} = -3x^3 - y, \qquad \frac{dy}{dt} = x^5 - 2y^3$$

21. Let $u(x)$ be any non-trivial solution of $u'' + q(x)\,u = 0$, where $q(x) > 0$ for all $x > 0$. Show that if $\int_1^\infty q(x)dx = \infty$,then u(x) has infinitely many zeroes on the positive x-axis.

22. State and prove sturm separation theorem.

23. Obtain Euler's differential equation for an extremal.

24. Find the point on the plane $ax + by + cz = d$ that is nearest the origin.

(7 x 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Find two independent Frobenius series solutions of the equation

$$2x^2y'' + xy' - (x+1)\,y = 0$$

26. Find the general solution of the system $\frac{dx}{dt} = 4x - 2y$, $\frac{dy}{dt} = 5x + 2y$

27. Determine Rodrigues formula for Legender polynomials and use it to find $P_0(x)$, $P_1(x)$, $P_2(x)$ *and* $P_3(x)$

28. Solve the initial value problem by picard's method $\frac{dy}{dx} = z, y(0) = 1$, $\frac{dz}{dx} = -y, z(0) = 0$

(2 x 4 = 8 Weightage)

****