19P201 (Pages: 2) Name……..……………….

Reg. No.....………………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCSS - PG)

CC19P MTH2 C06 - ALGEBRA II

(Mathematics)

(2019 Admissions - Regular)

Time: Three Hours Maximum: 30 Weightage

Part A (Short Answer questions**)**

Answer ***all*** questions. Each question carries 1 weightage.

1. Define Maximal ideal and Find all Maximal ideals of Z_6.
2. Does every algebraic extension is finite extension. Justify your answer.
3. Prove that set of all algebraic numbers forms a field.
4. Find all conjugates of $\sqrt{2} + i$ over Q.
5. Find number of isomorphisms from Q($\sqrt{2}$) to Q($\sqrt{3}$). Justify your answer.
6. Let σ be an automorphism of $Q(\pi)$ that map π onto – π. Find fixed field of σ.
7. Find $\Phi_8(x)$ over Q.
8. Show that the polynomial $x^7 - 1$ is solvable by radicals over Q.

(8 x 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Let R be a commutative ring with unity. Then prove that if M is a maximal ideal of R then R/M is a field.
10. Prove that trisecting the angle is impossible.
11. Let $E = Z_2(\alpha)$ be an extension field of Z_2 containing a zero α of $x^2 + x + 1$. Then write the addition table and multiplication table of E.

(2 x 2 = 4 Weightage)

UNIT II

12. If E is a finite extension of F, Then show that $\{E:F\}$ divides $[E:F]$
13. If K is a finite extension of E and E is a finite extension of F. Then show that K is separable over F if and only if K is separable over E and E is separable over F.
14. Define splitting field. Also find the splitting field of $x^4 - 2$ over Q.

(2 x 2 = 4 Weightage)

UNIT III

15. Describe the group of the polynomial $x^3 - 1$ over Q.

16. Show that $Q(\sqrt{5}, \sqrt{7})$ is a normal extension over Q.

17. Find Galois group of pth cyclotomic extension of Q for a prime p

(2 x 2 = 4 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. (a) Prove that the set of all constructible real numbers forms a subfield F of real numbers.

    (b) State and Prove Kronecker's Theorem.

19. (a) Define Perfect field. Prove that every field of characteristic zero is perfect.

    (b) State and Prove Primitive Element Theorem

20. (a) State and Prove Conjugation Isomorphism Theorem

    (b) If E is a splitting field over F. Then show that every irreducible polynomial in $F[x]$ having a zero in E splits in E.

21. (a) Let K be a finite normal extension of F , and let E be an extension of F. where $F \leq E \leq K$. Then prove that K is a finite normal extension of E and $G(K/E)$ is precisely the subgroup of $G(K/F)$ consisting of all those automorphisms that leave E fixed.

    (b) Let F be a field of characteristic 0 and let $a \in F$. If K is the splitting field of $x^n - a$ over F, then prove that $G(K/F)$ is a solvable group.

(2 x 5 = 10 Weightage)

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