19P204 (Pages: 2) Name……………………….

Reg. No……….….………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCSS-PG)

CC19P MTH2 C08 - TOPOLOGY -1

(Mathematics)

(2019 Admission: Regular)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** the questions. Each question carries 1 weightage.

1. Define scattering topology. Prove that every subset of irrational numbers is open in scattering topology.
2. Define sub-base of a topological space. Write a sub-base for R with usual topology.
3. Prove that the projection map is onto.
4. Give an example of a topological space which is Lindeloff but not compact.
5. Prove that metric spaces are first countable.
6. Define path connected space. Give an example.
7. Give an example of a topological space that is T_0 but not T_1.
8. Prove that in a Hausdorff space, limits of sequences are unique.

(8 x 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each of the following units.
Each question carries 2 weightage.

UNIT 1

9. Prove that the only convergent sequences in a discrete space are those which are eventually constant.
10. Prove that if a space is second countable then every open cover of it has a countable subcover.
11. Prove that for a subset A of a space , $\bar{A} = A \cup A'$.

UNIT 2

12. Prove that the product topology is the weak topology determined by the projection functions.
13. Prove that every separable space satisfies the countable chain condition.
14. Prove that every quotient space of a locally connected space is locally connected.

UNIT 3

15. Prove that every Tychonoff space is T_3.

16. Prove that every compact subset in a Hausdorff space is closed.

17. Prove that every Regular, Lindeloff space is normal.

(6 x 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. a) Define hereditary property of a topological space.

    b) Prove that metrisability is a hereditary property.

    c) Prove or disprove: Compactness is a hereditary property.

19. a) Prove that every continuous real-valued function on a compact space is bounded and attains its extrema.

    b) State and prove Lebesgue covering lemma.

20. a) Prove that a subset of R is connected iff it is an interval.

    b) Prove that metric spaces are T_4.

21. a) Prove that if a space X has the property that for any two mutually disjoint closed subsets A,B of it, there exist a continuous function $f: X \rightarrow [0,1]$ taking the value 0 at all points of A and the value 1 at all points of B, then X is normal.

    b) Let A be a closed subset of a normal space X and suppose $f: A \rightarrow [-1,1]$ is a continuous function. Then show that there exists a continuous function $F: X \rightarrow [-1,1]$ such that $F(x) = f(x) for\ all\ x \in A$.

(2 x 5 = 10 Weightage)

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