

19P205

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Name.....

Reg. No.....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCSS - PG)

(Mathematics)

CC19P MTH2 C09 - ODE AND CALCULUS OF VARIATIONS

(2019 Admissions - Regular)

Time: Three Hours

Maximum: 30 Weightage

Part A

Answer *all* questions. Each question carries 1 weightage.

1. Find the indicial roots of the equation $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$.
2. Find the general solution of $x(1 - x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ at $x = 0$.
3. Show that $\int_{-1}^1 \frac{1}{1-2xt+t^2} dx = \sum_{n=0}^{\infty} \left(\int_{-1}^1 [P_n(x)]^2 dx\right) t^{2n}$.
4. Show that $\frac{d}{dx} \{x^p J_p(x)\} = x^p J_{p-1}(x)$.
5. Compute Bessel series of the function $f(x) = 1$ in terms of the function $J_0(\lambda_n x)$.
6. Determine the nature of the critical $(0, 0)$ of the equation
$$\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + a^2x = 0, \text{ where } a \geq 0, b \geq 0 \text{ and } a = b.$$
7. Find the Extremal of the integral $\int_{x_1}^{x_2} [y^2 - (y')^2] dx$.
8. Find the Eigen values λ_n and Eigen functions $y_n(x)$ of $y'' + \lambda y = 0$ with $y(0) = y(\pi) = 0$.

(8 x 1 = 8 Weightage)

Part B

Answer any *two* questions from each unit. Each question carries 2 weightage.

UNIT I

9. Solve the differential equation $(1 - x^2)y'' - xy' + p^2y = 0$.
10. Discuss the nature at $x = \infty$ of the equation $x^2y'' + 4xy' + 2y = 0$ and find the exponents.
11. Derive Rodrigues's formula for $P_n(x)$.

UNIT II

12. Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

13. Find the critical point, differential equation, general solution and sketch the graph of

$$\begin{cases} \frac{dx}{dt} = ax - y \\ \frac{dy}{dt} = x + ay \end{cases}$$

14. Show that $(0, 0)$ is asymptotically stable critical point for the system $\begin{cases} \frac{dx}{dt} = -3x^3 - y \\ \frac{dy}{dt} = x^5 - 2y^3 \end{cases}$

UNIT III

15. Find normal form of the differential equation $y'' + p(x)y' + Q(x)y = 0$.

16. Find the exact solution of $y' = 2x(1 + y)$ with $y(0) = 0$. Starting with $y_0(x) = 0$ calculate $y_1(x), y_2(x), y_3(x)$ and $y_4(x)$, also compare these results with the exact solution.

17. State and prove Euler's differential equation for an Extremal.

(6 x 2 = 12 Weightage)

Part C

Answer any *two* questions. Each question carries 5 weightage.

18. Solve the differential equation $4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0$.

19. If there exists a Liapunov function $E(x, y)$ for the autonomous system, then the critical point $(0, 0)$ is stable. Furthermore, if this function has the additional property that the function $\frac{\partial E}{\partial x}F + \frac{\partial E}{\partial y}G$ is negative definite, then the critical point $(0, 0)$ is asymptotically stable.

20. Define Geodesies and find the Geodesies on a sphere $x^2 + y^2 + z^2 = a^2$.

21. State and prove Picard's theorem.

(2 x 5 = 10 Weightage)
