19P208S (Pages: 2) Name……………………

Reg. No……..…………..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCSS - PG)

(Supplementary/Improvement)

CC15P PHY2 C06 / CC17P PHY2 C06 - MATHEMATICAL PHYSICS - II

(Physics)

(2015 to 2018 Admissions)

Time: Three Hours Maximum: 36 Weightage

SECTION A

Answer ***all*** questions. Each question carries 1 weightage

1. Show that the real and imaginary parts of an analytic function are harmonic functions.
2. Define a pole of order m. How can be the residue of a pole of order m determined?
3. State and prove Cauchy's integral formula.
4. Can a group of order six have a subgroup of order four? Justify your argument.
5. If G $= \{A, A^2, A^3 = E\}$, find the element conjugate to A^2 .
6. Mention the features of a Lie group.
7. What idea does the Euler equation convey if x does not appear explicitly on the integrand i.e., $f = f(y, y_x)$?
8. Explain Rayleigh-Ritz variational technique.
9. How do you transform a second order ordinary differential equation into an integral equation?
10. Solve the integral equation $f(x) = \int_{-1}^{1} \frac{\varphi(t)}{(1-2xt+x^2)^{1/2}} dt, -1 \le x \le 1$, for the unknown function (t) , if f(x) = x^{2s}.
11. What are the properties of a 3-D Green's function?
12. Prove that Green's function is symmetric.

(12 x 1 = 12 Weightage)

SECTION B

Answer **any *two*** questions. Each question carries 6 weightages.

13. State and prove Cauchy residue theorem. Using the theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{a+bcos\theta}, a > b > 0$.
14. Show that the symmetry transformations of a square constitute a group. Also, determine its classes.

15. Derive Euler's equation by applying variational principle. How can it be generalized for the case of several independent variables?
16. Discuss the Neumann series technique to solve integral equations. Henceforth, evaluate the unknown function $\varphi(x) = 1 - \int_0^x (x-t)\varphi(t)dt$.

(2 x 6 = 12 Weightage)

SECTION C

Answer any ***four*** questions. Each question carries 3 weightages.

17. Evaluate the residue of $f(z) = \frac{\ln z}{(z^2+4)}$ at its poles.
18. By the method of contour integration, show that $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+4)^2} dx = \frac{\pi}{16}$.
19. Determine the multiplication table for the set of matrices:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}.$$

20. The ground state energy of a particle in a rectangular parallelopiped with sides a, b and c is given by $E = \frac{h^2}{8m}\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)$. Determine the shape of the box that will minimize the energy subject to the constraint that volume is a constant.
21. Derive the Fredholm integral equation corresponding to y"(x) = y(x), given y(1) = 1 and y(-1) = 1.
22. Find the eigen function expansion of Green's function for a harmonic oscillator problem.

(4 x 3 = 12 Weightage)

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