18P403S (Pages: 2) Name……………………

Reg. No…………………

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(supplementary)

(CUCSS-PG)

CC15P MT4 E02 - ALGEBRAIC NUMBER THEORY

(Mathematics)

(2015 - 2017 Admissions)

Time: Three Hours Maximum: 36 weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage

1. Express ${t_1}^4 + {t_2}^4$ in terms of elementary symmetric polynomials (n=2)
2. Find the order of the group $G/_H$ where G is a free abelian group with basis x, y, z and H is generated by $-2x, x+y, y+z$
3. Find θ such that $Q(\theta) = Q(\sqrt{2}, \sqrt[3]{5})$
4. Let x and y be nonzero elements of a domain. Prove that $x/_y$ iff $\langle x\rangle \supseteq \langle y\rangle$
5. Find a ring which is not noetherian.
6. State Minkowskis theorem.
7. Let d be a squarefree positive integer and let $k = Q(\sqrt{d})$. Calculate $\sigma : k \rightarrow L^{st}$
8. Let M be an R-module and N be an R-submodule of M. Show that the abelian group $M/_N$ has the natural structure of an R-module.
9. Let K be the number field Q(ξ) where ξ = $e^{2\pi i/p}$ for an odd prime p. If I is the ideal generated by λ = 1- ξ in the ring of integers $\mathbb{Z}[\xi]$ of K, then show that $I^{p-1} = < p >$ and N(I) = p.
10. Show that the only units of $Z[i]$ are $\pm 1, \pm i$
11. Prove that a prime in an arbitrary integral domain is always irreducible.
12. Define a regular prime and give an example of a regular prime.
13. Prove that every Euclidean domain is a principal ideal domain.
14. Prove that every maximal ideal is a prime ideal.

(14 x 1 = 14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Let G be a finitely generated abelian group with no nonzero elements of finite order. Prove that G must be a free group.

16. Let d be a square free rational integer with $d \not\equiv 1(mod\ 4)$. Then prove that $Z[\sqrt{d}]$ is the ring of integers of Q($\sqrt{d}$)

17. Prove that the group of units of Q($\sqrt{-3}$) is the group $\{\pm 1, \pm\omega, \pm\omega^2\}$, where $\omega = e^{\frac{2\pi i}{3}}$

18. Prove that a ring of integers of Q($\sqrt{-5}$) is not a unique factorization domain

19. If $\alpha_1, \alpha_2, \ldots, \alpha_n$ is a basis of the number field K over Q, then $\sigma(\alpha_1), \sigma(\alpha_2), \ldots, \sigma(\alpha_n)$ are linearly independent over R

20. Let G be a free abelian group with Z-basis $x_1, x_2, \ldots x_n$ and let H be a subgroup of G with Z-basis $y_1, y_2, \ldots, y_n$ with $y_i = \sum_j a_{ij}\, x_j$. Prove that $\left| {}^{G}/_{H} \right| = \left|\det(a_{ij})\right|$

21. Prove that the ring of integers of Q($\sqrt{-1}$) is norm Euclidean .

22. Show that an additive subgroup of R^n is a lattice if it is discrete.

23. Prove that factorization into irreducible is possible in a noetherian domain.

24. Compute the class number of Q($\sqrt{-6}$)

(7 x 2 = 14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Let $d < -11$ be a square free integer. Prove that the ring of integer of Q($\sqrt{d}$) is not Euclidean.

26. Let K be a number field. Then prove that there is an algebraic integer $\theta \in K$ such that K= Q($\sqrt{\theta}$)

27. Let D be a domain in which factorization into irreducible is possible. Prove that factorization into irreducible is unique iff every irreducible is prime.

28. Prove that the equation $x^4 + y^4 = z^2$ has no integer solution with $y, z \neq 0$

(2 x 4 = 8 Weightage)

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