18P450 (Pages: 2) Name: ………….……….

Reg. No……………………

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCSS - PG)

(Regular/Improvement/Supplementary)

CC15P ST4 C13 – MULTIVARIATE ANALYSIS

(Statistics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Show that $X \sim N_p(\mu, \Sigma)$ if and only if $T'X \sim N_1(T'\mu, T'\Sigma T)$ where T is any real vector.
2. If $X \sim N_p(0, I)$, then show that a quadratic form $X'AX$ and the linear form $B'X$ are independent if and only if $AB = 0$.
3. Distinguish between partial and multiple correlations.
4. Show that Wishart distribution is a matrix variate generalization of χ^2 distribution.
5. Show that every principal sub matrices of a Wishart matrix is again Wishart.
6. Define canonical correlation.
7. Show that Hotelling's T^2 statistic is invariant under non-singular transformation.
8. Establish the relation between Hotelling's T^2 and Mahalnobis D^2 statistics.
9. Write a short note on sphericity test.
10. What do you mean by Fisher's linear discriminant function?
11. How factor analysis and principal components are connected?
12. Explain how Fisher's discriminant function is related to Mahalnobis D^2.

(12 × 1 = 12 Weightage)

Part B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. Let $X = \begin{pmatrix} X^{(1)} \\ X^{(2)} \end{pmatrix}$ be a p-variate multivariate normal random vector. Obtain the necessary and sufficient condition for the independence of $X^{(1)}$ and $X^{(2)}$.
14. Show that $\bar{X}$ and S are independently distributed when sampling from a multivariate normal population.
15. Obtain the conditional distribution of X_1 given X_2 if (X_1, X_2) has a bivariate normal distribution with the parameters $\mu_1, \mu_2, \sigma_{1,}^2\sigma_2^2$ and ρ.

16. Obtain the MLE of $\sum$ when sampling from Multivariate Normal population with parameters 0 and $\sum$.
17. Derive the null distribution of the sample correlation coefficient.
18. Obtain the characteristic function of a Wishart distribution.
19. Explain the problem of symmetry based on T^2 statistic.
20. Describe how you will test the equality of covariance matrices of '*q*' multivariate normal distributions.
21. Derive the test criterion for testing independence of sub vectors of X where X follows $N_p(\mu,\Sigma)$.
22. Derive the Baye's procedure of classification into one of the two populations whose multivariate normal parameters are known.
23. Explain how the reduction in dimension is achieved through principal component analysis.
24. Describe the orthogonal factor model in Factor Analysis.

(**8 × 2 = 16 Weightage**)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. State and prove the Cochran's theorem for the independence of quadratic forms.
26. Derive the null distribution of the multiple correlation coefficient.
27. Explain multivariate Fisher-Behren problem.
28. Explain the role of eigenvalues and eigenvectors in Principal Component analysis. Describe an iterative procedure to calculate sample principal components.

(**2 × 4 = 8 Weightage**)

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