19U201S (Pages: 2) Name……………………

Reg. No……………………

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCBCSS – UG)

CC15U MAT2 B02/CC18U MAT2 B02 – CALCULUS

(Mathematics – Core Course)

(2015 to 2018 Admissions – Supplementary/Improvement)

Time: Three Hours Maximum: 80 Marks

Part 1 (Objective type questions)

Answer ***all*** questions. Each question carries 1 mark.

1. Define Point of inflection of a function.
2. Find the critical points of f(x) if $f'(x) = x\,(x-1)$.
3. What is the work done by a force $F(x) = 16 - \frac{4x}{5}$ lb along the x- axis from $x = 0$ to $x = 20$?
4. If $f = x^5 - 3x^4 + 9$, find df.
5. State the Max – Min Theorem for Continuous Functions.
6. Find the linearization of $f(x) = x^4$ at x = 1.
7. If P = {0,3,5,9,10 } is a partition of [0 ,10] then || P || = - - - - - - -
8. If f is continuous and $\int_1^3 f(x)\, dx = 6$, $\int_1^5 f(x)dx = 7$, then $\int_3^5 f(x)dx$ = - - - - -
9. If f is integrable on [a, b], its average (mean) value on [a, b] is - - - - - - - -
10. If $y = \int_0^x \sqrt{1+t^2}\,dt$ then $\frac{dy}{dx}$ = - - - - - -
11. Define a smooth function.
12. Give an example of a function which is not Riemann integrable**.**

(12 x 1 = 12 Marks)

Part II (Short answer type questions**)**

Answer any ***nine*** questions. Each question carries 2 marks.

13. Verify Rolle's Theorem for the function $f(x) = \frac{x^3}{3} - 3x$, in the interval $[-3, 3]$
14. Find the critical points of $f(x) = \frac{x^3}{3} - 2x^2 + 4x$, $0 \le x < \infty$.
15. Prove that the exponential function e^x is increasing on $\Re$.
16. Find the function f(x) whose derivative is 2x and whose graph passes through the point (-2, 3).
17. Evaluate $\lim_{x\to-\infty} \frac{\sqrt[3]{x} - 5x + 3}{2x + x^{\frac{2}{3}} - 4}$.
18. A 70 lb child and a 120 lb child are balancing on a seesaw. The 70 lb child is at 5 ft from the fulcrum. How far from the fulcrum is the 120 lb child?
19. Evaluate $\sum_{k=1}^{7}(-2k)$.

20. Use Max-Min Inequality to find upper and lower bounds for the value of $\int_0^1 \frac{1}{1+x^2} dx$.

21. Determine two positive numbers whose sum is 15 and the sum of whose squares is minimum.

22. Consider $f(x) = -x^2$, [0, 1]. Partition the interval into 4 subintervals of equal length and find the Riemann sum $\sum_{k=1}^{4} f(c_k)\, \Delta x_k$ where c_k is the left end point of each subinterval.

23. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\pi/4$.

24. Show that if f is continuous on [a, b], a≠b and if $\int_a^b f(x)dx = 0$ then f(x) = 0 at least once in [a,b].

(9 x 2 = 18 Marks)

Part III (Short essay or paragraph questions)

Answer any ***six*** questions. Each question carries 5 marks.

25. State and prove Mean Value Theorem for derivatives.

26. Find all asymptotes of the function $y = \frac{x^2-2}{x^2-1}$.

27. Find the average value of $f(x) = x^2 - 1$ on $[0,\sqrt{3}]$. At what point or points in the given interval does the function assume its average value?

28. Show that the center of mass of a straight thin strip or rod of constant density lies half way between its ends.

29. Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \; dt$.

30. Find the volume of the solid generated by revolving the region between the parabola $x = y^2+1$ and the line x = 3 about the line x = 3.

31. Prove that $\int_a^b x\, dx = \frac{b^2}{2} - \frac{a^2}{2}$, $0 < a < b$.

32. Estimate the sum of the square roots of the first n positive integers $\sqrt{1} + \sqrt{2} + - - - + \sqrt{n}$.

33. Find the area lying above x-axis, and included between the circle $x^2+y^2 = 2ax$ and the parabola $y^2 = ax$.

(6 x 5 = 30 Marks)

Part IV (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

34. Find the local maxima and minima of the function $f(x) = 2\cos 2x - \cos 4x$, $0 \le x \le \pi$, using second derivative test.

35. Graph the function $y = \frac{x^3 - 1}{x^2-1}$.

36. Find the area of the surface generated by revolving the curve $x = 2\sqrt{4-y}$, $0 \le y \le \frac{15}{4}$, about y-axis.

(2 x 10 = 20 Marks)

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