

19U202S

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Name: .....

Reg. No.....

**SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2020**

(CUCBCSS - UG)

**CC15U MAT2 C02 - MATHEMATICS**

Mathematics-Complementary Course

(2015 to 2018 Admissions - Supplementary/Improvement)

Time: Three Hours

Maximum Marks: 80

**Part I**

Answer *all* questions. Each question carries 1 mark.

1. Range of the function  $y = \cosh x$  is ...
2. Find  $\int \operatorname{sech}^2 x dx \dots$
3. Write the domain of the function  $w = \frac{1}{xy}$
4. If  $f(x, y) = 100 - x^2 - y^2$ , then the level set  $f(x, y) = 0$  is...
5. Define a smooth function.
6. What is the radius of the circle  $r = 6 \sin \theta$ .
7. The polar equivalent of the Cartesian equation  $xy = 2$  is...
8. Write an example of an infinite series which is convergent but not absolutely convergent.
9. Write the Maclaurin series expansion of  $e^x$ .
10. The  $n^{\text{th}}$  term of the sequence  $0, -\frac{1}{2}, \frac{2}{3}, -\frac{3}{4} \dots$  is ...
11. For the series  $\sum_{n=1}^{\infty} a_n$ , the  $n^{\text{th}}$  partial sum is...
12. Write an equation for the circular cylinder  $4x^2 + 4y^2 = 9$  in cylindrical co-ordinates.

(12 × 1 = 12 Marks)

**Part II (Short answer type )**

Answer any *nine* questions. Each question carries 2 marks

13. Given  $\sinh x = \frac{-3}{4}$ . Find the other five hyperbolic functions.
14. Find the derivative of  $y = \sinh^{-1} \sqrt{x}$
15. If  $f(x) = x + y + xy$ , find all second order partial derivatives.
16. Write the integral for the length of the curve  $y = x^2, -1 \leq x \leq 2$ .
17. The region between the curve  $y = x^2, 0 \leq x \leq 2$  and the  $x$ -axis is revolved about the  $x$ -axis to generate a solid. Find its volume.

18. Graph the set of points whose polar co-ordinates satisfy the conditions  $1 \leq r \leq 2$  and  $0 \leq \theta \leq \frac{\pi}{2}$ .
19. State the Continuous function theorem for sequences and using this prove that  $\sqrt{\frac{(n+1)}{n}} \rightarrow 1$ .
20. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{3^{n-1} - 1}{6^{n-1}}$ .
21. Find  $\frac{dy}{dx}$  if  $x^2 + \sin y - 2y = 0$ .
22. Find a polar equation for the hyperbola with eccentricity  $3/2$  and directrix  $x = 2$ .
23. Convert the polar equation  $r = \frac{4}{2\cos\theta - \sin\theta}$  into a cartesian equation.
24. Find a spherical equation for the cone  $z = \sqrt{x^2 + y^2}$ .

(9 × 2 = 18 Marks)

**Part III (Short essay type)**

Answer any **six** questions. Each question carries 5 marks marks.

25. If  $f(u, v, w)$  is differentiable and  $u = x - y$ ,  $v = y - z$  and  $w = z - x$ , then show that  $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$ .
26. Show that  $\sinh^{-1}x = \ln(x + \sqrt{x^2 + 1})$ ,  $-\infty < x < \infty$
27. Verify  $W_{xy} = W_{yx}$  if  $W = e^x + x \ln y + y \ln x$ .
28. Let  $a_1 = 1$  and let  $a_{n+1} = \frac{n}{2n+1}a_n$  for all  $n$ . Does the series  $a_n$  converge?
29. Identify the function  $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$ ,  $-1 \leq x \leq 1$
30. Find the Taylor series expansion of  $f(x) = \frac{1}{x^2}$  about  $x = 2$ .
31. Find the linearization of  $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$  at  $(3, 2)$ .
32. Evaluate
- (a)  $\int_0^{\ln 2} 4e^x \sinh x dx$                       (b)  $\int_0^{2\sqrt{3}} \frac{dx}{\sqrt{4+x^2}}$
33. Find the length of the curve  $y = (\frac{x}{2})^{2/3}$  from  $x = 0$  to  $x = 2$ .

(6 × 5 = 30 Marks)

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**Part IV (Essay type)**

Answer any **two** questions. Each question carries 10 marks

34. (a) Find the area of the surface generated by revolving the curve  $y = \sqrt{x}$ ,  $1 \leq x \leq 2$ , about the  $x$ -axis.
- (b) Find the volume of the solid generated by revolving the region between the parabola  $x = y^2 + 1$  and the line  $x = 3$  about the line  $x = 3$ .
35. (a) Discuss the convergence of the geometric series  $\theta, \theta r, \theta r^2, \dots, \theta r^{n-1}, \dots$
- (b) Find the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ .
36. (a) Find the area of the region that lies inside the circle  $r = 1$  and outside the cardioid  $r = 1 - \cos\theta$ .
- (b) Find the area of the surface generated by revolving the right-hand loop of the lemniscate  $r^2 = \cos 2\theta$ .

(2 × 10 = 20 Marks)

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