SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS -UG)

CC19U MTS2 C02 : MATHEMATICS II

(Mathematics - Complementary)

(2019 Admission - Regular)

Time: 2.00 Hrs   Max. Marks: 60

Credit: 3

(Draw diagram wherever necessary. The students can answer all questions in sections A & B)

A. Short answer questions. Each question carries 2 marks.

1. Find the slope of the tangent line of $r = \tan\theta$ at $\theta = \dfrac{2\pi}{3}$.

2. Evaluate $\displaystyle\int \sinh^2 x \quad dx$

3. Express the length of the graph of $f(x) = x\cos x$ on $[0, 1]$ as an integral.

4. State the comparison test for improper integrals.

5. Find $\displaystyle\lim_{n\to\infty} \dfrac{n^2 + 1}{3n^2 + n}$

6. Use Newton's method to find the first four approximations to a solution of the equation $x^2 = 4$, taking $x_0 = 1$.

7. Test for convergence of the series $\displaystyle\sum_{i=1}^{\infty} \dfrac{1}{1 + i^2}$

8. Show that the standard ordered basis for R^n is an orthonormal basis.

9. Suppose the system of equations $AX = B$ is consistent and A is a 5×8 matrix with $rank(A) = 3$. How many parameters does the solution of the system have?

10. Find rank of the matrix $\begin{pmatrix} 3 & -1 \\ 1 & 13 \end{pmatrix}$

11. Without solving determine whether the homogeneous system

$x_1 + x_2 - x_3 + x_4 = 0; \quad 5x_2 - 2x_4 = 0; \quad x_1 + x_3 - x_4 = 0; \quad 3x_1 + 2x_2 - x_3 + x_4 = 0$ has only the trivial solution or a nontrivial solution.

12. Diagonalize the matrix $\begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$.

(Ceiling: 20 Marks)

B. Short essay questions (Paragraph). Each question carries 5 marks.

13. Let $f(x) = x^3 - 4x^2 + 1$. Find an interval containing 1 on which f is invertible. Also find $f^{-1}(-7)$.

14. Find the area enclosed by the cardioid $r = 1 + \cos \theta$

15. Discuss the convergence of the p - series $\displaystyle\sum_{i=1}^{\infty} \frac{1}{i^p}$

16. Show that an alternating harmonic series converges conditionally.

17. Determine whether the set defined by $P(x) = c_3 x^3 + c_1 x$ is a subspace of the vector space P_3 , the set of all polynomials of degree less than or equal to 3.

18. Using properties of determinants evaluate $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

19. Compute A^m if $A = \begin{bmatrix} 8 & 5 \\ 4 & 0 \end{bmatrix}$. Then find A^5.

(Ceiling: 30 Marks)

C. Essay questions. Answer any one question.

20. Expand the function $f(x) = \dfrac{1}{(1 + x^2)}$ in a Maclaurin series. Use this to find $f''''''(0)$ and $f'''''''(0)$ without calculating derivative directly. Integrating the series of $f(x)$ prove that

$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$ for $|x| < 1$ and hence deduce $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

21. Find the eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix}$

(1 × 10 = 10 Marks)

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