18U404 (Pages: 3) Name: ……………………

Reg. No…………………..

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCBCSS - UG)

(Regular/Supplementary/Improvement)

CC15U MAT4 B04/ CC18U MAT4 B04

THEORY OF EQUATIONS, MATRICES AND VECTOR CALCULUS

(Mathematics - Core Course)

(2015 Admission onwards)

Time: Three Hours Maximum: 80 Marks

Part A

Answer ***all*** questions. Each question carries 1 mark.

1. State factor theorem.
2. Let α, β, γ be the roots of the equation $f(x) = 0$, then the roots of $f\left(\frac{1}{x}\right) = 0$ are -----------
3. Define reciprocal equation.
4. State Descarte's rule of signs.
5. Define rank of a matrix.
6. Let A be a 3x3 matrix whose rank is 2. Suppose that the elements of first row of A is multiplied by 4. What will be the rank of the new matrix?
7. State True/False. A homogeneous system is always consistent.
8. The eigenvalues of a triangular matrix are same as its -----------
9. Find parametric equations for the line through the origin and parallel to the vector $2\mathbf{j} + \mathbf{k}$.
10. Identify the surface $x^2 + y^2 + 4z^2 = 10$.
11. If $\mathbf{r}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + 6\,\mathbf{k}$, then $\lim_{t \to \frac{\pi}{4}} \boldsymbol{r}(t) =$ -----------
12. Find the unit tangent vector of the helix $\mathbf{r}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + t\,\mathbf{k}$.

(12 x 1 = 12 Marks)

Part B

Answer any ***nine*** questions. Each question carries 2 marks.

13. Solve the polynomial equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ of which one root is $-1 + i$.
14. By multiplying the roots by a suitable number transform the equation $9x^4 - 3x^3 + 2x^2 + 8x + 7 = 0$ into one with integral coefficient and the leading coefficient unity.
15. Remove the second term from the equation $x^3 - 6x^2 + 4x - 7 = 0$.
16. Show that the equation $x^6 + 3x^2 - 5x + 1 = 0$ has at least four imaginary roots.

17. Find the rank of $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ by reducing into echelon form.

18. Prove that the eigenvalues of a diagonal matrix is same as its diagonal elements.

19. Test for consistency the system of equations $x + 2y = 3, 2x + 4y = 7$.

20. If two of the eigenvalues of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ are 2 and 8, without expanding evaluate determinant of the matrix.

21. Find the point where the line $x = \frac{8}{3} + 2t,\ y = -2t,\ z = 1 + t$ intersects the plane

$$3x + 2y + 6z = 6.$$

22. Find a spherical coordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

23. The vector $\mathbf{r}(t) = (3\ \cos t)\ \mathbf{i} + (3\ \sin t)\ \mathbf{j} + t^2\ \mathbf{k}$ gives the position of a moving body at time t. Find the body's speed and direction when $t = 2$.

24. Show that $\boldsymbol{u}(t) = (\sin t)\ \boldsymbol{i} + (\cos t)\ \boldsymbol{j} + \sqrt{3}\ \boldsymbol{k}$ has constant length and is orthogonal to its derivative.

(9 x 2 = 18 Marks)

Part C

Answer any ***six*** questions. Each question carries 5 marks.

25. Prove that every polynomial of degree n has exactly n roots.

26. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, form an equation whose roots are $\alpha - \frac{1}{\beta\gamma}$, $\beta - \frac{1}{\gamma\alpha}$, and $\gamma - \frac{1}{\alpha\beta}$

27. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$

28. Reduce the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 7 & 13 \\ 4 & -3 & -1 \end{bmatrix}$ to its normal form and hence determine its rank.

29. Using elementary transformations find the inverse of the matrix $\begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$

30. Does the following system of equations possess a non-zero (i.e., non-trivial) solution?

$$x + 2y - 3z = 0; 2x - 3y + z = 0; 4x - y - 2z = 0.$$

31. Find the eigen values and find eigen vector corresponding to highest of the eigen value of the matrix A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

32. Find the distance from (1, 1, 3) to the plane $3x + 2y + 6z = 6$.

33. Show that the curvature of a circle of radius a is $\frac{1}{a}$

(6 x 5 = 30 Marks)

Part D

Answer any ***two*** questions. Each question carries 10 marks.

34. By reducing to standard form solve $x^3 - 6x^2 + 3x - 2 = 0$ by Cardano's method.

35. Verify Cayley-Hamilton theorem for the matrix A = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$ and then using the theorem obtain A^{-1} and A^{-2}.

36. Find **T**, **N**, **B** and κ for the space curve $\mathbf{r}(t) = (e^t\ \cos t)\ \mathbf{i} + (e^t\ \sin t)\ \mathbf{j} + 2\ \mathbf{k}$.

(2 x 10 = 20 Marks)

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