17U602 (Pages: 2) Name: ……………………

Reg. No……………………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2020

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT6 B10 - COMPLEX ANALYSIS

Mathematics - Core Course

(2015 Admission onwards)

Time: Three Hours Maximum:120 Marks

PART A

Answer ***all*** questions. Each question carries 1 mark.

1. What is the real part of z^2?
2. Give an example of an entire function.
3. $\frac{d}{dz}\cosh z =$ …………………………
4. Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic.
5. $e^{i\pi} =$…………………………
6. State Cauchy's integral formula.
7. Evaluate $\int_C \frac{z^2}{z-2} dz$ where C is the circle $|z| = 3$.
8. Define simply connected domain.
9. The radius of convergence of the series $1 + \frac{z}{1!} + \frac{z^2}{2!} + \cdots \frac{z^n}{n!} + \cdots$ is …………
10. What is isolated singular point?
11. Define pole of order m
12. Write the formula for Cauchy Principal Value.

(12 × 1 = 12 Marks)

PART B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Find a harmonic conjugate $v(xy)$ when $u(xy) = \frac{y}{x^2+y^2}$.
14. Find the principal values of $(-i)^i$.
15. Show that $f'(z)$ does not exists at any point of z when $f(z) = z - \bar{z}$.
16. Show that $|\sinh z|^2 = \sinh^2 x + \sinh^2 y$.
17. Find all values of z such that $e^z = 1 + i$.
18. Evaluate $\int_C \frac{2z^2+z}{z^2-1} dz$ where C is positively oriented circle $|z| = 1$.
19. If C is a simple closed contour containing the origin show that $\frac{1}{2\pi i}\int_C \frac{e^{az}}{z^{n+1}} dz = \frac{a^n}{n!}$.
20. State Laurent's theorem
21. Show that the sequence $z_n = -2 + i\frac{(-1)^n}{n^2}$ $(n = 1, 2, ..)$ converges to -2.
22. Find the Maclaurin series representation of $f(z) = e^z$.
23. Prove that absolute convergence of a series of complex numbers implies convergence of that series.
24. Find the pole of $f(z) = z^{-5}\sin z$

25. Using residue evaluate $\int_0^\infty \frac{dx}{x^2+1}$

26. Find the residue of the function $f(z) = \frac{z}{z^4+4}$ at the isolated singular point $z_0 = \sqrt{2}e^{i\pi/4}$

(10 × 4 = 40 Marks)

PART C

Answer any ***six*** questions. Each question carries 7 marks.

27. Find the derivative of the function $f(z) = \sqrt[3]{r}e^{\frac{i\theta}{3}}$ (r>0, $\alpha < \theta < \alpha + 2\pi$).
28. If a function f is analytic then show that it is independent of $\bar{z}$
29. Verify that $u(x, y) = y^3 - 3x^2y$ is harmonic in some domain. Also find its conjugate and the associated analytic function.
30. Find $\int_C z^2 dz$, where C is the line segment from $z = 0$ to $z = 2 + i$
31. Evaluate $\int_C \frac{z^2 - \frac{1}{3}}{z^3 - z} dz$ where C is the circle $\left|z - \frac{1}{2}\right| = 1$ oriented in the counter clockwise direction.
32. If a function f is analytic at a given point then show that its derivatives of all orders are analytic at there too
33. Show that $\frac{e^z}{z(z^2+1)} = \frac{1}{z} + 1 - \frac{1}{2}z - \frac{5}{2}z^2 + \cdots (0 < |z| < 1)$
34. Expand in Laurent series of $\frac{1}{z(z-1)^2}$ at the point $z = 1$.
35. Show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi(a+2b)}{2ab^3(a+b)^2}$, $a > 0$, $b > 0$

(6 x 7 = 42 Marks)

PART D

Answer any ***two*** questions. Each question carries 13 marks.

36. State and prove maximum modulus principle.
37. Find series representations of $f(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{z-1} - \frac{1}{z-2}$.
38. Show that $\int_0^{2\pi} \frac{\sin^2\theta d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$ $a > b > 0$

(2 x 13 = 26 Marks)

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