17U604 (Pages: 2) Name: ………….……….

Reg. No…………….………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION APRIL 2020

(CUCBCSS-UG)

(Regular/Supplementary/Improvement)

CC15U MAT6 B12 - NUMBER THEORY AND LINEAR ALGEBRA

Mathematics - Core Course

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

Part- A (Objective Type)

Answer ***all*** questions. Each question carries 1 mark.

1. For any integer a, prove that $a|0$.
2. Define Euclidean numbers.
3. Write a solution of the equation $2x \equiv 3(mod 5)$
4. Translate the decimal number 74 in binary form.
5. Write the remainder when 16! is divided by 17
6. Find the highest exponent of 2 that divides 11!
7. Find the sum of the divisors of 180
8. Write a proper subspace of the space $\mathbb{R}^2$
9. Write the dimension of the space of polynomials of degree ≤ 2 over $\mathbb{R}$
10. Define the linear transformation between two vector spaces
11. Give a vector space isomorphic to $\mathbb{R}^2$
12. Write a basis of the space of 2×2 matrices over the field $\mathbb{R}$

(12 × 1 = 12 Marks)

Part-B (Short Answer Type)

Answer any ***ten*** questions. Each question carries 4 marks.

13. Prove that $5^{2n} + 7$ is divisible by 8 for all $n \geq 1$
14. If $a|bc$, with $gcd\ (a, b) = 1$, then prove that $a|c$.
15. Find all prime numbers that divide 50!
16. Find the remainder when $712! + 1$ is divided by 719.
17. Find the smallest number with 10 numbers
18. Prove that $\varphi(n)$ is an even integer for $n > 2$
19. Prove that $\varphi(p^k) = p^k - p^{k-1}$, where p is a prime number and $k \geq 1$
20. For any prime p, $\tau(p!) = 2\tau((p-1)!)$
21. Prove that union of two subspaces of a vector space is need not be a subspace.
22. Let V is a vector space over the field F. Prove that $\alpha O = O$, where $\alpha \in F$ and O is the zero vector in V.

23. Is $\mathbb{Q}$ over $\mathbb{R}$ a vector space? Justify your answer.
24. Write $(1,0)$ as a linear combination of $(1,1)$ and $(-1,2)$ in $\mathbb{R}^2$
25. Show that $f:\mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = (x^2, y)$ is not a linear map.
26. Show that $S = \{1, x, x^2, x^3\}$ is a basis of $\mathbb{R}_3[x]$.

(10 × 4 = 40 Marks)

Part-C (Short Essay Type)

Answer any ***six*** questions. Each question carries 7 marks.

27. Prove that for given integers a and b, not both of which are zero, there exist integers x and y such that $gcd(a,b) = ax + by$.
28. Prove that the greatest common divisor of two positive integers divides their least common multiple.
29. Divide 100 into two summands such that one is divisible by 7 and the other by 11, using the concept of Diophantine equation.
30. If p_n is the n^{th} prime number, then prove that $p_n \geq 2^{2^{n-1}}$
31. Solve the linear congruence $17x \equiv 9(mod\ 276)$, using Chinese Remainder Theorem.
32. Assume that p and q are distinct odd primes. Then prove that $p^{q-1} + q^{p-1} \equiv 1(modpq)$
33. Prove that the set of 2×2 matrices over the real field is a vector space.
34. Let $f: U \to V$ be a linear map. Prove that if X is a subspace of U, then $f^{\rightarrow}(X)$ is a subspace of V.
35. If V is an n-dimensional vector space over the field F, then prove that V is isomorphic to F^n.

(6 × 7 = 42 Marks)

Part- D (Essay Type)

Answer any ***two*** questions. Each question carries 13 marks.

36. State and prove Euler's Theorem. Deduce Fermat's little theorem.
37. Prove that the linear Diophantine equation $ax + by = c$ has solution if and only if $d|c$, where $d = gcd(a,b)$. Explain the method to obtain all the solutions of the Diophantine equation.
38. Let $f: \mathbb{R}_2[x] \to \mathbb{R}^2$ be a map defined by $f(a + bx + cx^2) = (a - b, b - c)$. Prove that f is a linear map. Find range, null space, rank and nullity of f. Verify the dimension theorem.

(2 × 13 = 26 Marks)

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